OPTIMAL SCHEDULING IN HYDRO-THERMAL POWER SYSTEMS BY THE METHOD OF LOCAL VARIATIONS

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Dedicated

to

my parents

CERTIFICATE

Certified that this work, 'Optimal Scheduling in Hydro-Thermal Power Systems by the Method of Local Variations', by Mr. Kallury Surya Prakasa Rao has been carried out under our supervision and that this work has not been submitted elsewhere for a degree.

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$x_i(k)$	=	Storage in the i-th hydro reservoir at the beginning of the k-th subinterval
$u_i(k)$	=	Discharge rate from the i-th hydro reservoir during the k-th subinterval
$P_{Hi}(k)$	=	Power generated by the i-th hydro station in the k-th subinterval
$P_{Ti}(k)$	=	Power generated by the i-th thermal station during the k-th subinterval
$L_i(k)$	=	Inflow rate into the i-th hydro reservoir during the k-th subinterval
$e_i(k)$	=	Evaporation rate from the i-th reservoir during the k-th subinterval.
$F_i(P_{Ti}(k))$	=	Fuel cost of generating power of P_{Ti} MW during the k-th subinterval at the i-th thermal station
$P_D(k)$	=	Load demand on the system in MW in the k-th subinterval
H_{oi}	=	Basic head of the i-th hydro reservoir (defined when the reservoir is empty)
c_i	=	Correction factor for head variation in any subinterval k, at the i-th hydro station
G	=	Constant used in the determination of hydro power generated in MW
N	=	The number of equal subintervals into which the entire interval of scheduling is partitioned
$P_i(k)$	=	Active power produced at the bus i in the k-th subinterval
$Q_i(k)$	=	Reactive power produced at bus i in the k-th subinterval

$C_i(k)$ = Active power consumed at bus i in the k-th subinterval

$D_i(k)$ = Reactive power consumed at bus i in the k-th subinterval

$I_i(k)$ = Active power injection at bus i in the k-th subinterval

$K_i(k)$ = Reactive power injection at bus i in the k-th subinterval

$V_i(k)e^{j\delta_i(k)}$ = Complex bus voltage at the i-th bus in the k-th subinterval

$Y_{ij}e^{-j\theta_{ij}}$ = $G_{ij}-jB_{ij}$ = ij-th element of the bus impedance matrix [Y]

G_{ij} = Conductance of the ij-th element of the bus admittance matrix

B_{ij} = Susceptance of the ij-th element of the bus admittance matrix

$x_{i\ min}$ = Lower bound on the storage of the i-th hydro reservoir

$x_{i\ max}$ = Upper bound on the storage of the i-th hydro reservoir

$P_{i\ min}$ = Minimum allowable power generation at the i-th bus

$P_{i\ max}$ = Maximum allowable power generation at the i-th bus

$Q_{i\ min}$ = Minimum allowable reactive power generation at the i-th bus

$Q_{i\ max}$ = Maximum allowable reactive power generation at the i-th bus

$S_{i\ max}$ = Maximum allowable apparent power at the i-th bus

$V_{i\ min}$ = Minimum allowable voltage at the i-th bus

$V_{i\ max}$ = Maximum allowable voltage at the i-th bus

$u_{i\,min}$ = Lower bound on the discharge rate of the i-th hydro reservoir

$u_{i\,max}$ = Upper bound on the discharge rate of the i-th hydro reservoir

$T_{ij\,max}$ = Upper bound on the voltage phase angle difference between buses i and j, with i and j being neighbouring buses

MLV = Method of Local Variations

IDP = Incremental Dynamic Programming

k = Index of the time instant or the subinterval

$*$ = Optimal values.

SYNOPSIS

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OPTIMAL SCHEDULING IN HYDRO-THERMAL POWER SYSTEMS BY THE METHOD OF LOCAL VARIATIONS

In an interconnected power system, it is possible to supply a given load demand in many ways, and hence it is natural for the operator to look for the 'best' or 'optimum' operating strategy. Thus optimal scheduling in a power system implies the determination of a strategy which would optimize stipulated operating criterion. A particular choice of this criterion in power system operation is the cost of the fuel consumed and the objective is to minimize the same. Early work in this area was that of Kirchmayer, who solved a purely thermal system problem, using the coordination equations derived by the Lagrange multiplier technique. In this formulation, the effects of the equality and inequality constraints imposed by the transmission and generating systems were either approximated or completely neglected. Carpentier in 1962 formulated this purely thermal problem more rigorously, taking into consideration the several equality and inequality constraints of the system, which

were not considered by Kirchmayer. Subsequently several solution techniques were proposed in the literature to solve the problem as formulated by Carpentier. However, the interest in this thesis is directed towards a power system which consists of thermal as well as hydro (including pumped storage) plants. The optimal scheduling of such a system is different from a purely thermal system in the following aspects. No fuel cost is associated with hydro stations and the total amount of water that is available for power generation over a specified period is limited. Hence the solution to this problem at any given time consists of determination of a plan for withdrawal of water from the hydro reservoirs for power generation and determination of the corresponding thermal generations so that the total cost of fuel is minimized. At the same time, the total power demand on the system is met and the operational constraints such as limits on the reservoir storages, the rates of discharges from the reservoirs, the power generations at various units, the voltages and phase angles in the network, the reactive power generations, and the line flows stipulated by stability considerations etc. are satisfied. The dynamics of the hydro system makes this a variational problem.

Several attempts have been made in the past to solve the above problem by dynamic programming,incremental

dynamic programming, and several indirect optimization methods like the discrete maximum principle or the continuous maximum principle. In all these attempts, the difficulty encountered was that of enormous computational requirements for a problem of realistic size. Therefore, this investigation is aimed at finding an algorithm, which is simple, easy to implement, and requires less computational effort and storage. A direct search method known as the 'Method of Local Variations' (MLV), (due to Krylov and Chernous Ko') is found to meet the above requirements due to its simplicity, efficiency, and ease of implementation. The essential features of this method are a discrete-time description of the system, a decomposition of the combined system into hydro and thermal subsystems, a starting nominal trajectory of hydro generations, and a systematic iterative method of perturbing the nominal trajectory such that the total fuel cost is monotone decreasing from iteration to iteration till a satisfactory convergence to an optimal trajectory is obtained. At every stage of the algorithm, all the constraints of the problem are satisfied.

In this thesis, two mathematical models of the hydro-thermal system are considered, pertaining to (1) a short range scheduling problem, and (2) a long range scheduling problem. In the short range model, the electrical network of the power system is represented

in detail. This enables consideration of various engineering and operating constraints on voltages, reactive powers and line flows etc. for the entire scheduling period. In the long range problem, however, only an approximate solution is sought. Instead of representing the electrical network in detail, the transmission losses in the system are approximated by Kirchmayer's loss coefficients. No other electrical constraints like those on voltages, reactive powers etc. are considered. Ofcourse in both the formulations, constraints on hydro storages, discharge rates and power generations are considered.

To illustrate the applicability and advantages of the MLV, first a one-hydro-one-thermal problem similar to the one solved by Bernholtz and Graham (using incremental dynamic programming method) has been solved. Next a more complex long range problem consisting of three hydro and four thermal plants has been solved. The computational advantages of the MLV approach are established.

Next a short range problem consisting of two hydro and two thermal plants, similar to the one considered by Bonaert, El-Abiad and Koivo has been solved. It is shown that the MLV applied to this problem is superior to the method used by Bonaert etal.

A modification of the basic MLV approach, which consists in varying the stepsize in the algorithm is presented and it is shown that this can further reduce the computational effort significantly.

Finally the optimization problem which includes pumped storage hydro plants is also considered. It is shown that the MLV approach permits this situation to be considered without any major modifications. Numerical results are presented to illustrate the method.

CHAPTER 1

INTRODUCTION

1.1 GENERAL INTRODUCTION TO THE PROBLEM

With the advent of high speed digital computers, it has become possible to conduct optimal planning and scheduling studies of large interconnected power systems in a rigorous manner using sophisticated mathematical models for the various components of the power system. In an interconnected power system with various power sources, more than one operating strategy is possible to meet the power demand in the system and hence it is natural for the operator to seek an optimum operating strategy. This is determined by optimizing a pre-specified operating criterion with reference to some control variables in the power system. An important criterion in a power system is the cost of the fuel consumed at the generating stations and it is to be minimized. This problem of optimal scheduling or economic load dispatch has been the concern of the operating engineers over the last two decades. Kirchmayer's [1] early work on economic dispatching in a purely thermal power system stimulated considerable interest in this area. His method consists of essentially solving a set of coordination equations formulated using the Lagrange multipliers technique.

The solution determines the most economic generation levels for a set of thermal generating units of unequal efficiency feeding into a lossy transmission system and constrained to satisfy the power demand. The transmission loss in the system is approximately considered using the loss coefficients [2]. It was Carpentier [3] who pointed out in 1961, that Kirchmayer's formulation was incomplete in the sense that the equality and inequality constraints imposed by the transmission and generation systems were either approximated or completely disregarded. Furthermore, the reactive power injection which controls the voltage level in the network was omitted in this formulation. Carpentier [3] in 1962 formulated this problem as a nonlinear programming problem, wherein the transmission network is modelled by stipulating that the equality constraints defined by the power flow equations be satisfied. Also the voltages, phase angles, and real and reactive injections are required to satisfy the specified inequality constraints. The function to be minimized in this case is the fuel cost at the generating stations in the system. Several iterative techniques such as SUMT and gradient techniques [4,5,6] have been developed in the literature to solve the above nonlinear programming problem.

However, a modern power system may consist of a large number of thermal, conventional hydro and pumped storage power plants connected to various load centres

through a lossy transmission network. An important objective in the operation of such a power system is to generate and transmit power to meet the system demand at minimum fuel cost by an optimal mix of the various types of plants. The study of the problem of optimum scheduling of power generation at various plants in a power system is of paramount importance in a large country like India, where most state electricity boards depend for their power requirements on both hydro and thermal power generations and the power systems are going through a phase of rapid expansion.

Hydro-thermal optimal scheduling problem is a more complex one than the purely thermal system optimization problem discussed above since the hydro reservoir dynamics and the constraints on the reservoir storages and withdrawals of water from the reservoirs have also to be taken into account. In this problem the cost of hydro power generation (fuel cost) can normally be neglected in comparison with the cost of thermal power generation. However, the water that can be used over a prescribed interval is limited by the capacity, and inflows into the reservoirs, prespecified withdrawal from the reservoirs for meeting agricultural, navigational and interstate requirements etc. The procedure for integrating the operation of hydro and thermal generation in a hydro-thermal system for minimum cost of generation has

been referred to as hydro-thermal coordination [2]. The solution to this economic or optimal load scheduling problem for hydro-thermal systems consists of determination of a plan for withdrawal of water from the reservoirs for power generation, and the corresponding thermal generations so as to minimize the total cost of thermal generations over a specified period, while meeting the total power demand on the power network. The dynamics of the hydro system makes this a variational problem. Further, the resulting schedule has to satisfy the operational constraints such as limits on the reservoir storages, discharge-rate constraints, limits on voltages and phase angles, limits on active and reactive generations, and limits on the power transfers along the individual lines etc. These constraints are primarily imposed on the variables in order that the equipment ratings and the desired operating conditions of the generators and the electrical network are not violated.

Due to the exhibition of cyclic nature, by the inflows into the reservoirs and the load demand on the power system, the problem of optimal scheduling in hydro-thermal power systems can be broadly classified into long and short range problems. The long range problem is normally defined over an year and the short range problem over a day or a week. A search into the literature reveals that the basic difference between these two problems is the manner in which the

characteristics of the power system network are represented in the mathematical model used in the analysis. In a long range problem, the transmission losses in the system have been approximately considered as a quadratic function of the power generations in the system, using the loss coefficients due to Kirchmayer [2]. The operating constraints on voltages, reactive power generations, the line flows etc. in the power network are not considered. But in a short range problem the transmission losses in the system are considered by the accurate solution of the nonlinear power flow equations of the power network. In this formulation, the operational constraints on the voltages, reactive powers, the line flows etc. are duly incorporated.

The proper integration of hydro and thermal generation in a power system of realistic size is quite complex and presents considerable difficulty in the determination of the optimal hydro and thermal schedules. Many algorithms have been proposed in the past for this problem. In this thesis another algorithm is proposed which the author believes requires less computational effort and storage than the existing ones. Any such claim can ofcourse be established only after a comparative study is made of the existing and the proposed algorithms bringing out their relative advantages and disadvantages. Some effort is made towards this direction in the present work, however it is difficult to do due to lack of sufficient information regarding data and numerical results in the literature.

1.2 STATE OF THE ART OF THE PROBLEM

Several attempts have been made in the past to solve the optimal scheduling problem for hydro-thermal power systems using direct as well as indirect search techniques such as Dynamic Programming, Pontryagin's maximum principle etc. A detailed survey of the more important solution methods used for this problem solution is given in [7]. A brief overview of the state of the art of the above problem is given below.

Dandeno, Chandler and Kirchmayer [8] assumed that the hydro plants operate at essentially constant head and formulated a short range problem with loss coefficients as follows. Let $F(P_{Tj}(t))$ be the cost of generation in unit time for a generation of P_{Tj} units at the j-th thermal generating station. Let $W_i(t)$ be the discharge rate at the i-th hydro station for a generation of P_{Hi} units and let K_i be the total volume of water available for power generation. Let $P_R(t)$ be the load demand at any time t and let $P_{Loss}(t)$ be the transmission losses expressed as a function of thermal and hydro generations using loss coefficients. Then it is desired to

$$\text{minimize} \quad \int_0^{t_1} \{ \sum_{j=1}^{\alpha} F(P_{Tj}(t)) \} dt \tag{1.1}$$

with the restriction that

$$\int_0^{t_1} W_i(t)\, dt = K_i \, , \qquad i=\alpha+1,\ldots,\beta \tag{1.2}$$

and

$$\sum_{j=1}^{\alpha} P_{Tj}(t) + \sum_{i=\alpha+1}^{\beta} P_{Hi}(t) - P_{Loss}(t) = P_R(t) \quad (1.3)$$

where α = number of steam plants

$\beta-\alpha$ = number of hydro plants

t_1 is the total interval of optimization. By the application of variational methods, the following coordination equations are derived

$$\frac{dF_j(P_{Tj}(t))}{dP_{Tj}(t)} + \lambda \frac{\partial P_{Loss}(t)}{\partial P_{Tj}(t)} = \lambda \quad (1.4)$$

$$\gamma_i \frac{dW_j(t)}{dP_{Hi}(t)} + \lambda \frac{\partial P_{Loss}(t)}{\partial P_{Hi}(t)} = \lambda \quad (1.5)$$

where

$\frac{dF_j(P_{Tj}(t))}{dP_{Tj}(t)}$ = incremental production cost of steam plant j

$\frac{\partial P_{Loss}(t)}{\partial P_{Tj}(t)}$ = incremental transmission loss of steam plant j

$\frac{\partial P_{Loss}(t)}{\partial P_{Hi}(t)}$ = incremental transmission loss of hydro plant i

$\frac{dW_j(t)}{dP_{Hi}(t)}$ = incremental water rate at hydro plant i

λ = incremental cost of received power

γ_i = water conversion coefficient which converts incremental water rate into equivalent incremental plant cost

$$P_{Loss}(t) = \sum_{j=1}^{\alpha} \sum_{i=\alpha+1}^{\beta} P_j(t) B_{ji} P_i(t) \tag{1.6}$$

B_{ji} = loss coefficients.

Equations (1.2) to (1.5) when solved give the solution, for $P_{Tj}(t)$ and $P_{Hi}(t)$ over the entire interval. The values γ_i which effectively convert incremental water rates into incremental plant costs, determine the amounts of water used at each hydro plant. Therefore γ_i must be so chosen that the desired amounts of water are utilized, i.e. eqn.(1.2) is satisfied. λ is so chosen that the load demand is always satisfied in eqn.(1.3). It is observed that γ_i's are constants and λ is a function of time.

When the number of hydro plants to be coordinated increases, a choice of γ conversion factors, which will result in desired water storage becomes very difficult. Similarly the choice of λ will also be difficult as computational instability was observed by Dandeno [9] while iterating on λ for the loads at certain hours. Dandeno [9] further observed that by direct application of the coordination equations, solutions are obtained which sometimes dictate generations outside the plant capacities, because these constraints are not included in the problem formulation.

Bernholtz and Graham [10] to [13] considered a short range problem with loss coefficients and attempted to solve it through Incremental Dynamic Programming (IDP), procedure. This method consists essentially of applying the Dynamic Programming algorithm locally, in a neighbourhood of a nominal trajectory. Starting from an initial feasible trial schedule, the problem of determining optimum schedule for a system consisting of any number of constant head hydro stations and any number of steam stations, was reduced to a sequence of problems involving, first, the determination of hydro-schedules which maximize the weighted output of the hydro system and second the determination of the corresponding minimum cost thermal schedules. With increased number of hydro stations in the system, this method becomes much involved as the computational requirements become enormously high.

Sokkappa [14] formulated the long range problem (see Sec. 2.1) as a nonlinear programming problem. Transmission losses were considered through the use of loss coefficients. For each subinterval of optimization, the constraint, which is most likely to be violated was picked up and a slack variable associated with this constraint. The gradient of the cost function was evaluated and the steepest descent method used to obtain the solution of the problem, starting from a known initial schedule. Such a procedure becomes impossible in view of large dimensional requirements for problems of multi-reservoir hydro-thermal problems.

Dynamic Programming was used by Petersen [15] to solve an annual optimization problem. Hano etal [16] and Dahlin and Shen [17] have described computational approaches to the economic operation of a multi-reservoir hydro-thermal system employing the continuous maximum principle of Pontryagin. Narita [18] and Oh [19] have employed the discrete maximum principle to solve the same problem. In all these methods, the computational requirements for a realistic size problem become prohibitively high.

Bonaert etal [20] have employed the method of solution given by Bernholtz and Graham [10] in the framework of a more sophisticated model. By a suitable decomposition of the total system into a hydro subsystem and a thermal subsystem, their method consists of starting with a feasible nominal hydro schedule, the thermal subsystem is solved for optimal thermal generations, by obtaining optimal power flow solutions [6] one for each subinterval. The incremental costs of hydro powers known as dual variables are also obtained from the optimal power flow solutions. Using these dual variables, a new neighbourhood hydro schedule is obtained in such a way that the overall cost function is minimized in each iteration. The iterative process is continued till the overall minimum cost of operation is obtained. As evidenced by the example given in [20],

it appears that this method also suffers from the large computational requirements for practical size systems.

Gopala Rao [21] has formulated the short range problem as an additively separable nonlinear programming problem and used Lasdon's decomposition [22] technique, to split the problem of larger dimension into subproblems of smaller dimensions to be solved iteratively. The main difficulty in this method is the initial choice of the dual variable vector, which plays a dominant role in the computational time required.

1.3 OUTLINE OF THE THESIS

The central theme of this work is the application of the Method of Local Variations [23], [24] (MLV), to obtain solutions to the complex problem of optimal scheduling in hydro-thermal power systems. A brief outline of the material discussed in the following chapters is given below.

The second chapter introduces the two basic approaches which arise due to the repetitive nature of the system data, like the load demand on the system, the inflows into the reservoirs etc. This leads to the classification of the problem into short range and long range problems. Justification of the consideration of the transmission losses in both the cases is attempted. Two mathematical models have been formulated defining the problem of optimal scheduling in hydro-thermal power

systems, one for the short range problem and the other for the long range problem. The main difference in these two models is the manner in which the electrical network is represented. A brief discussion is also included to bring out the salient features for and against both the models described in this chapter.

Chapter 3 briefly describes the computational techniques used in the solution procedures adopted in later chapters. The Method of Local Variations in the context of its application to the work reported is described in detail. Next the Augmented Penalty Function approach of Hestenes [25] as applied to mathematical programming problems by Miele etal [26] is discussed briefly. Optimal Power Flow solution procedure of Dommel and Tinney [6] is then briefly summarized. The material in this chapter is aimed at providing a good introduction to the basic features of the solution techniques employed.

In Chapter 4, the long range model is considered. Solution to this model is obtained using the Method of Local Variations. A numerical example is given to illustrate the solution procedure for a one-hydro-one-thermal problem. In this case a nominal hydro schedule is assumed to start with and the corresponding thermal generations are obtained by solving a quadratic equation of power balance condition for each subinterval.

The nominal hydro schedule is perturbed in its neighbourhood using the MLV iteratively to obtain the optimum schedule. A comparison between the MLV procedure and the method due to Bernholtz and Graham [10] is given for the above problem. The method has then been extended to solve a multi hydro, multi thermal problem. In this case, knowing the nominal hydro generations at each hydro plant determined from the nominal trajectory, it is necessary to solve a sequence of mathematical programming problems one for each subinterval. Both equality and inequality constraints have been taken into account in solving the mathematical programming problems using the Augmented Penalty Function approach adopted by Miele etal [26] and the modifications given in Sec. 3.3. A numerical example of a power system consisting of three hydro and four thermal plants is solved and the results are presented. A recommended procedure for the choice of the step size vector, while working with the MLV is made based on the computational experience gained during the course of this work.

In Chapter 5, a short range problem is considered. In this case a detailed mathematical model is employed, wherein the transmission losses are accounted for by solving the A.C. power flow equations during each scheduling subinterval. In each subinterval, knowing the hydro generations at all the hydro plants, an optimal

power flow solution [6] is obtained for solving the optimal thermal generations. While obtaining the optimal power flow solution, the nominal hydro generations are considered as injections into the power network at the buses at which the hydro plants are situated. Thus starting with a nominal trajectory of hydro schedules, a sequence of optimal power flow solutions are to be computed one for each subinterval. The nominal trajectory is then improved iteratively by perturbing it in its neighbourhood by using the MLV. This ensures a monotone decrease of the cost of the thermal generations in the system, from iteration to iteration. A numerical example illustrates the procedure. A brief comparison of this method with that of Bonaert etal [20] is also given.

Chapter 6 shows the generality of the MLV, by applying it to solve a power system optimal scheduling problem consisting of conventional as well as pumped storage hydro plants. Numerical example illustrates the method.

The seventh chapter gives an overall summary of the work reported in this thesis and a few suggestions or possibilities for further research in this area are given.

CHAPTER 2

PROBLEM DEFINITION

2.1 INTRODUCTION

The problem of optimum scheduling in hydro-thermal power systems can be broadly classified into long and short range problems. The long range problem refers normally to an annual problem. The inflows into hydro reservoirs exhibit an annual cyclicity. Furthermore, there may be a seasonal variation in power demand on the system, and this too exhibits an annual cyclicity. The optimization interval of one year duration is thus a natural choice for long range optimal generation scheduling studies. The solution to the scheduling problem in this case consists of determination of the amounts of water to be drawn from the reservoirs for hydro-generation in each scheduling subinterval and the corresponding thermal generations to meet the load demand over each subinterval, utilizing the entire amount of water available for power generation during the total interval. It is necessary in this problem to predict beforehand the load demand, inflows into and evaporations from the hydro-reservoirs for the entire optimization year. In this case the load demand on the system is probabilistic in nature. For the purpose of scheduling studies normally forecasts are prepared from the past load data of the system using

statistical prediction methods. Reservoir dynamics should be considered in the formulation of the problem [27] since there may be significant variation in reservoir head in the optimization period. The normal scheduling subintervals for this problem are a day, a week or even a month and average values for hydro and thermal generations in these subintervals are determined in the long range problem. Due to very nature of the problem rigorous power network model representation is not necessary in this case. It is sufficient to consider the transmission losses in the system using loss coefficients developed by Kirchmayer [2].

The short range problem usually has an optimization interval of a day or a week. This period is normally subdivided into hourly subintervals for scheduling purposes. The solution to the long range problem will stipulate the amounts of water to be utilized over each day or week (the scheduling subinterval of the long range problem). Then the solution to the short range problem consists of an optimal plan for utilization of this water for power generation and the corresponding optimal thermal generations determined, considering the load demand on the system and the constraints imposed on its operation. In this case, it is the normal practice to assume that the load demand and the inflow data for the

reservoirs are known deterministically. Approximate determination of transmission losses by utilizing the loss coefficients, as is done in a long range scheduling problem, may not yield schedules sufficiently close to the optimal ones. Furthermore the loss coefficients [2] are determined for a given system network configuration as a function of the generated powers in the network. Also there is no way of checking the operating limits on the voltage magnitudes in the network and the reactive power generations at their sources. However for the practical implementation of the optimum schedule, it is necessary to incorporate the constraints on the electrical network such as the upper and lower limits on the bus voltages, the maximum and minimum allowable reactive power generations by the reactive power sources, and the limits imposed on the phase angle difference between any two adjacent buses for stability considerations. Thus the transmission losses are therefore usually included exactly by solving the nonlinear power network equations. The head variations in reservoirs during the optimization interval of a short range problem are usually insignificant and can be neglected. There may however be situations where this variation has to be taken into account by considering the reservoir dynamics. The above two models of the optimal hydro-thermal scheduling problem are formulated in Secs.2.2 and 2.3.

2.2 MATHEMATICAL MODEL FOR LONG RANGE SCHEDULING

A power system consisting of 'h' hydro plants and 'r' thermal plants is considered. A discrete-time representation of the continuous time optimal scheduling problem is presented here. The total interval of optimization is divided into N equal subintervals, subinterval being assumed to be each of unit length for simplicity. The hydro stations are assumed to be operating with reservoirs which are independent of each other. It is also assumed that the load demand $P_D(k)$, reservoir inflows $L_i(k)$ and evaporation losses $e_i(k)$, $(k=1,\ldots,N)$, over the optimization interval are known accurately enough in advance. The initial and final reservoir storages represented by the vectors $\underline{X}(1)$ and $\underline{X}(N+1)$ respectively are specified. These values represent the constraint on the amount of water to be consumed over the optimization interval. The state equations representing the hydro system dynamics are

$$x_i(k+1) = x_i(k) + L_i(k) - u_i(k) - e_i(k), \quad (i=1,\ldots,h), \quad (k = 1,\ldots,N) \tag{2.1}$$

The optimization problem is to determine the rates of discharges $u_i(k)$, $(k = 1,\ldots,N)$, $(i = 1,\ldots,h)$ and the thermal generations $P_{Tj}(k)$, $(k = 1,\ldots,N)$, $(j = 1,\ldots,r)$ such that the performance index, representing the total thermal power generation cost,

$$J = \sum_{k=1}^{N} \sum_{j=1}^{r} F_j(P_{Tj}(k)) \tag{2.2}$$

is minimized subject to the equality constraint

$$P_D(k) = \sum_{j=1}^{r} P_{Tj}(k) + \sum_{i=1}^{h} P_{Hi}(k) - P_{Loss}(k) ,$$

$$(k = 1,\ldots,N) \qquad (2.3)$$

and the following inequality constraints

$$u_{i\ min} \leq u_i(k) \leq u_{i\ max} , \ (i = 1,\ldots,h), \ (k = 1,\ldots,N) \qquad (2.4)$$

$$x_{i\ min} \leq x_i(k) \leq x_{i\ max} , \ (i = 1,\ldots,h), \ (k = 2,\ldots,N) \qquad (2.5)$$

$$P_{Tj\ min} \leq P_{Tj}(k) \leq P_{Tj\ max} , \ (j = 1,\ldots,r), \ (k = 1,\ldots,N) \qquad (2.6)$$

The power balance equation (2.3) states that the total power generated should be equal to the sum of total load demand and transmission losses in any subinterval k. Equations (2.4) to (2.6) specify the upper and lower bounds on the rates of water discharge, water storage, and thermal generations in each subinterval. In eqn.(2.2), the function F_j yields the fuel cost for thermal generation of the j-th station and is considered to be a quadratic of the form

$$F_j(P_{Tj}(k)) = a_j P_{Tj}(k) + b_j P_{Tj}^2(k) \qquad (2.7)$$

where a_j and b_j are constants determined from the fuel cost-output characteristics of the j-th thermal station. In eqn.(2.3), $P_{Loss}(k)$, the transmission loss during the k-th subinterval, is expressed, following Kirchmayer [2], in terms of hydro and thermal power generations as

$$P_{Loss}(k) = \sum_{i=1}^{h+r} \sum_{j=1}^{h+r} P_i(k)\, B_{ij} P_j(k), \quad (k = 1,\ldots,N) \qquad (2.8)$$

where

$$P_i(k) \triangleq P_{Hi}(k), \quad (i = 1,\ldots,h),$$

$$P_i(k) \triangleq P_{Tj}(k), \quad (i = h+1,\ldots,h+r), (j = 1,\ldots,r) \qquad (2.9)$$

The loss coefficients B_{ij} are assumed to be available for the given power system. The hydro generation at the i-th station during the k-th subinterval can be related to the water head, the storage of the reservoir at the beginning and end of the interval, and the discharge rate by a relation [16] of the form

$$P_{Hi}(k) = \frac{H_{0i}}{G}[1 + \frac{c_i}{2}(x_i(k) + x_i(k+1))]u_i(k),$$

$$(i = 1,\ldots,h), \quad (k = 1,\ldots,N) \qquad (2.10)$$

where c_i and G are known constants.

Thus the problem is to determine the drawdown at each hydro station and the generation at each thermal station over the optimization interval such that the water available for hydro generation is fully utilized, the equality and inequality constraints specified in eqns.(2.3) and (2.4) to (2.6) are satisfied and the fuel cost in thermal generation is minimized.

2.3 MATHEMATICAL MODEL FOR SHORT RANGE SCHEDULING

As in Section 2.2, a discrete version of the continuous time optimal scheduling problem is employed in the formulation. An electric power system of 'n' buses

with 'h' hydro stations and 'r' thermal stations is considered. The total interval of optimization is divided into N equal subintervals, each of which is assumed to be of unit length for simplicity. The load in each subinterval is assumed to remain constant at all load buses. It is also assumed here, without loss of generality, that the power at a specific bus is generated either by a thermal or by a hydro plant, but not by both. The hydro stations are assumed to be operating with reservoirs which are independent of each other. The initial and final storages of the reservoirs are specified. These values, which specify the amount of water available for power generation during the interval of optimization, are generally obtained from the solution of the long range problem.

We will assume here for the sake of generality that the reservoir head variations are not negligible. Thus the hydro system equations remain the same as in Sec. 2.2 and are repeated here for the sake of continuity. State equations describing the hydro system dynamics therefore, are

$$x_i(k+1) = x_i(k) + L_i(k) - u_i(k),$$
$$(i = 1,\ldots,h),\ (k = 1,\ldots,N) \qquad (2.11)$$

Here the evaporation losses term is omitted, since in a short range problem the evaporation effect is usually negligible. The hydro generation during the k-th

subinterval at the i-th station can be related to the water head, discharge rate and the subinterval terminal storages by a relation [16] of the form

$$P_{Hi}(k) = \frac{H_{0i}}{G}[1 + \frac{c_i}{2}(x_i(k) + x_i(k+1))]u_i(k) \ ,$$
$$(i = 1,\ldots,h),\ (k = 1,\ldots,N) \qquad (2.12)$$

Bounds on the volumes and the rates of discharge of water from the reservoirs are given by

$$u_{i\ min} \leq u_i(k) \leq u_{i\ max}\ ,\ (i = 1,\ldots,h),\ (k = 1,\ldots,N) \qquad (2.13)$$

$$x_{i\ min} \leq x_i(k) \leq x_{i\ max}\ ,\ (i = 1,\ldots,h),\ (k = 2,\ldots,N) \qquad (2.14)$$

The formulation of the thermal system and the electrical network follows closely the Carpentier-Sirouxs' formulation as given by Peschon [28]. During any subinterval k, the solution to the thermal system and the electrical network can be viewed as a static optimization problem under the assumption that the generations and the loads during this subinterval are constant.

The cost of power generation in the system is considered to depend only on the active powers $P_{Ti}(k)$, generated by thermal plants. Thus the performance index J, representing the total power generation cost

$$J = \sum_{k=1}^{N} \sum_{i=1}^{r} F_i(P_{Ti}(k)) \qquad (2.15)$$

is to be minimized subject to the following equality and inequality constraints.

Equality Constraints:

Equality constraints arise due to the requirement of real and reactive power balance at each bus and for each subinterval. Thus for a typical bus i, we have the following equations.

The real power balance is given by

$$I_i(k) - P_i(k) + C_i(k) = 0 \ , \ (i = 1,\ldots,n), \ (k=1,\ldots,N) \tag{2.16}$$

and the reactive power balance is given by

$$K_i(k) - Q_i(k) + D_i(k) = 0 \ , \ (i = 1,\ldots,n), \quad (k = 1,\ldots,N) \tag{2.17}$$

where

$$I_i(k) = \sum_{\substack{j=1 \\ j\neq i}}^{n} V_i(k)V_j(k)Y_{ij}\text{Cos}(\Theta_{ij} + \delta_i(k) - \delta_j(k)) + V_i^2(k)\ Y_{ii}\text{Cos}\ \Theta_{ii},\ (k = 1,\ldots,N) \tag{2.18}$$

$$K_i(k) = \sum_{\substack{j=1 \\ j\neq i}}^{n} V_i(k)V_j(k)Y_{ij}\text{Sin}(\Theta_{ij} + \delta_i(k) - \delta_j(k)) + V_i^2(k)Y_{ii}\text{Sin}\ \Theta_{ii}\ ,\ (k = 1,\ldots,N) \tag{2.19}$$

It may be observed that to get the expressions for I_i and K_i, the model for the transmission line is taken to be an equivalent π network.

Inequality Constraints:

The related variables $P_i(k)$, $Q_i(k)$, and $V_i(k)$ are subjected to upper and lower limits, which are

primarily determined by the equipment ratings and operating conditions of the generators and the electrical network.

$$P_i^2(k) + Q_i^2(k) - S_{i\ max}^2 \leq 0 \tag{2.20}$$

$$P_{i\ min} - P_i(k) \leq 0 \tag{2.21}$$

$$Q_{i\ min} - Q_i(k) \leq 0 \tag{2.22}$$

$$Q_i(k) - Q_{i\ max} \leq 0 \tag{2.23}$$

$$V_{i\ min} - V_i(k) \leq 0 \tag{2.24}$$

$$V_i(k) - V_{i\ max} \leq 0 \tag{2.25}$$

The restrictions on the power transfers along the lines for ensuring system stability can be represented by

$$|\delta_i(k) - \delta_j(k)| \leq |T_{ij\ max}| \tag{2.26}$$

Thus the problem consists of minimizing (2.15) subject to equality constraints (2.16) and (2.17) and inequality constraints (2.20) to (2.26). Also the constraints imposed on each reservoir storage and discharge rates (2.12) and (2.13) have to be met.

2.4 DISCUSSION

The first model defined in Sec.2.2 relates to a long range problem and in this model the transmission losses are approximately considered using the loss coefficients [2]. Further, there is no reflection of the characteristics of the electrical network in this model and hence there is no

provision to adhere to the operating constraints on the bus voltages, reactive powers, and the line flows. The model is simple and the computational requirements in solving the problem will be quite less. But the solution on implementation may result in inadmissible voltage solutions at the buses in the transmission system and stability requirements stipulated in terms of the maximum phase difference across a particular line may not be met. However, this model is very useful to solve a long range problem which provides a good starting data base for the short range problem, which uses a rigorous and sophisticated model as defined in Sec.2.3.

The model defined in Sec. 2.3 is applicable for a short range problem. Since in this formulation, the limits on the bus voltages and phase angles, reactive powers, and line flows are considered, the solution obtained will always be admissible. However, for this rigorous and sophisticated model, the solution methods are more complex and require large computational effort.

2.5 SUMMARY

In this chapter, the problem of optimal scheduling in hydro-thermal power systems has been mathematically defined following the two basic approaches of considering the characteristics of the electrical network. In the first model, which is normally used for the solution of

a long range problem, the transmission losses are approximately considered, but the characteristics and constraints on the transmission network are totally ignored. Thus the solution obtained may not be always feasible. The second model is formulated for a short range problem. The transmission losses are exactly included accounting for both active and reactive components. The various operating limits on the bus voltages, reactive powers, and the line flow requirements are considered in this case. The solution thus obtained is always feasible and implementable. A brief comparison of the above two models is given from their applicability and limitations considerations.

CHAPTER 3

SOLUTION TECHNIQUES

3.1 INTRODUCTION

To solve the optimal hydro-thermal scheduling problem, the Method of Local Variations [23], [24], the Augmented Penalty Function Method [26] and the Optimal Power Flow Solution [6], have been employed after suitable modifications. The algorithms as used in the solution are briefly discussed below.

3.2 METHOD OF LOCAL VARIATIONS (MLV)

The Method of Local Variations (MLV) was developed by Chernous' Ko [23] and Krylov and Chernous' Ko [24] for the numerical solution of optimal control problems. The method was also used by Singaraj [29] in solving optimal structural design problems. The major advantage of this method is its simplicity and its efficacy to deal with the various boundary conditions and inequality constraints on both the state and control variables. The essential features of this method are a discrete time description of the system, a starting nominal trajectory, and a systematic iterative method of perturbing the nominal trajectory, such that the cost index (objective function)is monotone decreasing in successive iterations till a satisfactory convergence to the optimal trajectory is obtained.

Statement of the Problem:

Consider the discrete time version of a continuous time control process described by the system of difference equations

$$\underline{X}(k+1) = g(\underline{X}(k), \underline{u}(k), k), \quad (k = 1, \ldots, N) \qquad (3.1)$$

where $\underline{X}(k)$, and $\underline{u}(k)$ are state and control vectors of dimensions h and m respectively. N is the number of discrete time subintervals into which the total interval of optimization is divided. It is required to find a phase trajectory $x_i(k)$, $(i = 1, \ldots, h)$, $(k = 1, \ldots, N+1)$ and its control $u_i(k)$, $(i = 1, \ldots, m)$, $(k = 1, \ldots, N)$, such that (3.1) is satisfied and the functional

$$J(\underline{X}, \underline{u}) = \sum_{k=1}^{N} F_k(\underline{X}(k), \underline{u}(k), k) \qquad (3.2)$$

is minimized where $F_k(\underline{X}(k), \underline{u}(k), k)$ is the cost incurred in the k-th subinterval, subject to the constraints on the state and control variables given by

$$\underline{u}(k) \in U(k), \quad (k = 1, \ldots, N) \qquad (3.3)$$

$$\underline{X}(k) \in G(k), \quad (k = 1, \ldots, N+1) \qquad (3.4)$$

where U(k) and G(k) are generally variable closed regions of h and m-dimensional spaces respectively. The initial and final conditions for the vector $\underline{X}(k)$ are taken care of by properly defining the regions G(1) and G(N+1). The solution of the above problem using the Method of Local Variations will be clear from the algorithm described below.

Algorithm Description:

An initial nominal trajectory $\underline{X}^1(k)$, $(k = 1,\ldots,N+1)$, is considered to start with. This initial trajectory need not be a feasible trajectory. Feasibility here means that the nominal trajectory does not violate the state constraints and the corresponding control variables also do not violate the constraints imposed on them. A survey of the initial trajectory at the instants $k = 1,\ldots,N+1$, reveals at what instants the constraints (3.3) and/or (3.4) are not satisfied. If for some k, $(1 \leq k \leq N+1)$, either (3.3) or (3.4) are not satisfied, the initial nominal trajectory is not feasible and hence is not suitable. In this case, the initial nominal trajectory is modified (subcontrolled) by changing the unsatisfactory $\underline{X}^1(k)$. This change consists of adding one after another to the unsatisfactory components $x_j^1(k)$, of the vector $\underline{X}^1(k)$, a quantity $r\Delta x_j$ where the numbers j, r take on the values $j = 1,\ldots,h$, $r = \pm 1,\ldots,\pm s$. $\Delta x_j > 0$ are the given magnitudes of the steps in the coordinates and s is a given integer. If this method of variation permits revision of $\underline{X}^1(k)$, which satisfies the constraints (3.3) and (3.4), the survey of the successive points is carried out. If the initial nominal trajectory cannot be corrected by this variational (subcontrol) operation, at any instant, the initial trajectory is discarded and a different initial approximation is selected, which may yield a feasible nominal trajectory

to start with. Here the values of the cost functional $J^1(\underline{X}^1, \underline{u}^1)$ corresponding to $\underline{X}^1(k)$, $(k = 1,\ldots,N+1)$ as well as the subinterval costs are computed and stored.

After obtaining an initial nominal trajectory, which is feasible, in the second part of the algorithm, the current nominal state trajectory $\underline{X}^i(k)$, $(k = 1,\ldots,N+1)$ in the i-th iteration is varied as described below to obtain the nominal trajectory for the (i+1)-th iteration. The variation is carried out successively for each of the components of $\underline{X}^i(k)$; initially for the first one, then for the second etc. For each component $x_r(k)$, its own step of variation $\Delta x_r(k) > 0$ is chosen. Let the initial and final conditions on the states i.e., $\underline{X}(1)$ and $\underline{X}(N+1)$ be assumed to be known or specified. Then it is to be noted that the initial and final values of the nominal state trajectories always satisfy the specified conditions. Now the following local variations operation is performed on the nominal state trajectory at time instants $k = 2,\ldots,N$. Let it be assumed that variation of the state values at the k-th instant is under consideration, and also let $x_j^i(k)$ be under consideration. The upper index gives the iteration number and the lower index gives the number of the component of the state vector. Completion of the variation of all the components of the state vector $\underline{X}(k)$, $(k = 2,\ldots,N)$, signifies the end of an iteration. At the beginning of the variation of the j-th component of the vector $\underline{X}(k)$,

all the preceding components have already been varied; i.e., the state vector at the k-th instant is $(x_1^{i+1}(k),\ldots,x_{j-1}^{i+1}(k),\ x_j^i(k),\ldots,x_h^i(k))^T$. Here $x_1^{i+1}(k)$, $\ldots,x_{j-1}^{i+1}(k)$ represent the new values assigned to the first(j-1) components of the state vector in the i-th iteration by the application of the MLV. Variation of the j-th component of the state vector at instants $\ell = 1,\ldots,k-1$, have already been performed so that the current state trajectory is $x_r^{i+1}(\ell)$, $(r = 1,\ldots,j-1)$, $(\ell = 2,\ldots,N)$; $x_j^{i+1}(\ell)$, $(\ell = 2,\ldots,k-1)$, $x_j^i(\ell)$, $(\ell = k,\ldots,N)$; $x_r^i(\ell)$, $(r = j+1,\ldots,h)$, $(\ell = 2,\ldots,N)$. Now the value of the component state at the k-th instant is varied to $x_j^i(k) + \Delta x_j(k)$,keeping all the other state values unchanged. Tests are performed to see whether (1) this state value is admissible, (2) the control inputs required to translate the state from $\underline{X}(k-1)$ to $\underline{X}(k+1)$ via $\underline{X}(k)$ are admissible (where the states $\underline{X}(k-1)$, $\underline{X}(k)$ and $\underline{X}(k+1)$ are defined in the above trajectory), (3) the changed performance index value for the subintervals (k-1) and k taken together is less than the corresponding performance index value for the same two sub-intervals for the trajectory, before the j-th component at the k-th instant is varied. If the answer is in the affirmative for the above three tests, then the j-th component of the state vector at the k-th instant is changed to $x_j^{i+1}(k) = x_j^i(k) + \Delta x_j(k)$. If the answer is in the negative, the same three tests are performed for

$x_j^i(k) - \Delta x_j(k)$. If this proves successful the j-th component of the state at the k-th instant is changed to $x_j^{i+1}(k) = x_j^i(k) - \Delta x_j(k)$. Otherwise the old value of the component of the state $x_j^i(k)$ is retained at this instant and the local variation operation is performed on $x_j^i(k+1)$ at the instant (k+1), and so on. When the variation of the j-th component at all instants $k = 2,\ldots,N$ is completed, the variation process is shifted to (j+1)-th component of the state and so on. In this way, the local variation operation is applied successively for all components of the state vector at all instants of time to complete one iteration and obtain the nominal trajectory for the (i+1)-th iteration. These iterations, which ensure a decrease in the values of the cost function in each iteration, are performed till a satisfactory convergence to an optimum has been obtained. The resulting $\underline{X}^*(k)$, $(k = 1,\ldots,N+1)$ and $\underline{u}^*(k)$, $(k = 1,\ldots,N)$ are the desired optimal state and control vectors respectively.

3.3 AUGMENTED PENALTY FUNCTION METHOD

Considerable work has been done over the past several years on the solution of minimization problem with equality constraints, which may be stated as

$$\text{minimize } f = f(\underline{x}) \tag{3.6}$$

subject to the constraints

$$\underline{\phi}(\underline{x}) = \underline{0} \tag{3.7}$$

Here, f is a scalar function of $\underline{x}$, $\underline{x}$ an n-dimensional vector, $\underline{\phi}$ is a q-dimensional vector with $q < n$. It is assumed that the first and second partial derivatives of f and $\underline{\phi}$ with respect to $\underline{x}$ exist and are continuous.

Any $\underline{x}_o$ which satisfies constraints (3.7), is said to be a feasible solution and the set of all the feasible solutions is called a feasible set,

$$T \triangleq \{\underline{x} | \ \underline{\phi}(\underline{x}) = 0\} \qquad (3.8)$$

$\underline{x}^*$ is said to be an optimal solution to the problem if the following is true

$$f(\underline{x}^*) \leq f(\underline{x}) \quad \text{for every } \underline{x} \in T \qquad (3.9)$$

The usual method of solution for the above nature of problem has been the method of Lagrange Multipliers. Hestenes [25] modified it by augmenting the function by a penalty function in addition to the usual terms obtained by the product of Lagrange Multipliers and the equality constraints (Augmented Penalty Function Method). It has been explored by Miele etal [26] in connection with the ordinary first order gradient algorithm. The method usually exhibits faster convergence than the ordinary Lagrange Multiplier method [26]. A brief description of the steps in the algorithm is given below.

From the theory of maxima and minima, the problem defined by eqns. (3.6) and (3.8) can be recast as that of minimizing the augmented function

$$F(\underline{x}, \underline{\lambda}) = f(\underline{x}) + \underline{\lambda}^T \ \underline{\phi}(x) \qquad (3.10)$$

Here $\underline{\lambda}$ is a q-vector Lagrange multiplier and the superscript T denotes the transpose of a vector or a matrix. If

$$\underline{F}_{\underline{x}}(\underline{x}, \underline{\lambda}) = \underline{f}_{\underline{x}}(\underline{x}) + \phi^T_{\underline{x}}(\underline{x})\, \underline{\lambda} \qquad (3.11)$$

denotes the gradient of the augmented function, the optimum solution $(\underline{x}^*, \underline{\lambda}^*)$ must satisfy the simultaneous equations

$$\underline{\phi}\,(\underline{x}^*) = \underline{0} \qquad (3.12)$$

$$\underline{F}_{\underline{x}}(\underline{x}^*, \underline{\lambda}^*) = \underline{0} \qquad (3.13)$$

Eqn.(3.13) is known as the optimum condition. Now in the Augmented Penalty Function Method, a new function $W(\underline{x}, \underline{\lambda}, R)$ is defined as follows

$$W(\underline{x}, \underline{\lambda}, R) \triangleq F(\underline{x}, \underline{\lambda}) + \underline{\lambda}^T \underline{\phi}(\underline{x}) + R\, P(\underline{x}) \qquad (3.14)$$

where $$P(\underline{x}) \triangleq \underline{\phi}^T(\underline{x})\, \underline{\phi}(\underline{x}) \qquad (3.15)$$

Here the scalar $R > 0$ is the penalty constant. It was suggested by Miele etal [26], that faster convergence is possible if the Lagrange multiplier and the penalty constant are chosen at each iteration so as to obtain certain desirable properties, for instance (a) descent property on the augmented penalty function, (b) descent property on the augmented function, (c) descent property on the constraint error, and either (d) constraint satisfaction on the average or (e) individual constraint satisfaction. Properties (d) and (e) are employed to first order only.

The Lagrange multiplier $\underline{\lambda}$ is determined by minimizing the error in the optimum condition defined in eqn.(3.16) with respect to $\underline{\lambda}$ for given $\underline{x}$. Owing to the fact that the error in the optimum condition is

$$Q(\underline{x},\underline{\lambda}) \triangleq [\underline{f}_{\underline{x}}(\underline{x}) + \phi^T_{\underline{x}}(\underline{x})\,\underline{\lambda}]^T[\underline{f}_{\underline{x}}(\underline{x}) + \phi^T_{\underline{x}}(\underline{x})\underline{\lambda}] \qquad (3.16)$$

the Lagrange multiplier vector is derermined by the relation

$$\underline{Q}_{\lambda}(\underline{x},\ \underline{\lambda}) = 0 \qquad (3.17)$$

which yields

$$\phi^T_{\underline{x}}(\underline{x})\ \phi_{\underline{x}}(\underline{x})\ \underline{\lambda} + \phi^T_{\underline{x}}(\underline{x})\ \underline{f}_{\underline{x}}(\underline{x}) = \underline{0} \qquad (3.18)$$

This linear vector equation is equivalent to q linear scalar relations in each of which the only unknown is the Lagrange multiplier and can easily be determined. Similarly the penalty constant R is chosen in such a way that on the average, the constraints are satisfied to first order [30] and is given by

$$R = 2P(\underline{x})/\underline{P}^T_{\underline{x}}(\underline{x})\ \underline{P}_{\underline{x}}(\underline{x}) \qquad (3.19)$$

Thus the Lagrange multiplier $\underline{\lambda}$, and the penalty constant R are determined in each iteration as explained above. The basic algorithm is summarized as follows.

The Basic Algorithm:

Let $\underline{x}$ denote the nominal point, $\underline{\tilde{x}}$, the varied point and $\Delta\,\underline{x}$ the displacement leading from the nominal point to the varied point. Let $\underline{\lambda}$ denote the Lagrange

multiplier, R, the penalty constant, and α the gradient step size. The gradient algorithm is represented by

$$
\begin{aligned}
\underline{F}_{\underline{x}}(\underline{x}, \underline{\lambda}) &= \underline{f}_{\underline{x}}(\underline{x}) + \phi^T_{\underline{x}}(\underline{x})\,\underline{\lambda} \\
\underline{P}_{\underline{x}}(\underline{x}) &= 2\phi_{\underline{x}}(\underline{x})\,\underline{\phi}\,(\underline{x}) \\
\underline{W}_{\underline{x}}(\underline{x}, \underline{\lambda}, R) &= \underline{F}_{\underline{x}}(\underline{x}, \underline{\lambda}) + R\,\underline{P}_{\underline{x}}(\underline{x}) \qquad (3.20) \\
\Delta\underline{x} &= -\alpha\,\underline{W}_{\underline{x}}(\underline{x}, \underline{\lambda}, R) \\
\tilde{\underline{x}} &= \underline{x} + \Delta\underline{x}
\end{aligned}
$$

For a given nominal point $\underline{x}$, Lagrange multiplier $\underline{\lambda}$, and penalty constant R, equations (3.20) constitute a complete iteration leading to the varied point $\tilde{\underline{x}}$ for a specified gradient step size α.

In the above algorithm it is assumed that $\underline{x}$ can assume any value. But if in addition to the equality constraints given by eqn.(3.7), there are inequality constraints on $\underline{x}$ of the form

$$
\underline{x}_{\min} \leq \underline{x} \leq \underline{x}_{\max} \qquad (3.21)
$$

Then the inequality constraints eqn.(3.21) can be handled easily by assuring that the adjustment algorithm in (3.20) does not send any component of $\underline{x}$ beyond its permissible limits. If the correction Δx_i from (3.20) would cause x_i to exceed its limits, $\tilde{x}_i$ is set to the corresponding limit.

$$
\tilde{x}_i = \begin{cases} x_{i\,max}\ , & \text{if} \quad x_i + \Delta x_i > x_{i\,max} \\ x_{i\,min}\ , & \text{if} \quad x_i + \Delta x_i < x_{i\,min} \\ x_i + \Delta x_i & \text{otherwise} \end{cases} \tag{3.22}
$$

Even when a component of $\underline{x}$ has reached its limit, its component in the gradient vector must still be computed in the following cycles because it might eventually back off from the limit.

In the present work, while working with the long range problem discussed in Sec.2.2,it is required to solve a sequence of mathematical programming problems one for each subinterval to solve for the optimal thermal generations, knowing the hydro generations from the nominal trajectory. In each subinterval k, the cost of the thermal generation (2.2) is to be minimized subject to the equality constraint (2.3), (power balance equation) and the inequality constraints (2.6). The above method of Augmented Penalty Function is employed to solve the mathematical programming problem encountered in each subinterval (Ref. Chapter 4).

3.4 OPTIMAL POWER FLOW SOLUTION

In Sec. 2.3, the model for short range optimization problem in hydro-thermal power systems was described. It can be seen that the problem in each subinterval once again consists of minimizing a cost function (2.15) subject to the equality constraints (2.16) and (2.17) (power flow equations) and the inequality constraints (2.20) to (2.26).

It can be seen that for a power system of n buses, usually a set of 2(n-1) equality constraints will result. We could employ the method of Augmented Penalty Function discussed in the previous section also for this problem. However, the Optimal Power Flow Solution due to Dommel and Tinney [6] which is based on simple Lagrange Multiplier Method would be more suitable here and will become clear from the description given below.

Equations (2.16) and (2.17) generally will give rise to a set of 2(n-1) equations corresponding to an n-bus system (the equations for the slack bus would not be needed). Any solution which satisfies the above equations constitutes a power flow solution. Dommel and Tinney [6] solve the power flow problem by Newton's method and then the adjustment of the control parameters so as to minimize the objective function is done by the gradient method. The optimization problem developed depends on the assumption that initially the inequality limits are not incorporated. The problem of optimal power flow can thus be stated as

$$\underset{[\underline{u}]}{\text{minimize}}\ f(\underline{x}, \underline{u}) \tag{3.23}$$

where $f(\underline{x}, \underline{u})$ is the objective function and $\underline{x}$ and $\underline{u}$ are dependent and control variable vectors respectively. Any bus in a power system network is characterized by four quantities, namely the magnitude of the voltage at

the bus, its corresponding phase angle, the real and reactive powers entering the bus of which two of them are specified and the other two are to be found. This will give rise to the dependent and control variables. The minimization in eqn.(3.23) is subject to equality constraints

$$\underline{g}(\underline{x}, \underline{u}) = \underline{0} \tag{3.24}$$

Using the Lagrangean formulation, the above problem is equivalent to minimizing the unconstrained Lagrangian

$$L(\underline{x},\underline{u}) = f(\underline{x},\underline{u}) + \underline{\lambda}^T \underline{g}(\underline{x},\underline{u}) \tag{3.25}$$

$\underline{\lambda}$ is the Lagrange multiplier vector. The following necessary conditions are to be satisfied for a minimum.

$$\underline{\frac{\partial L}{\partial \underline{x}}} = \underline{\frac{\partial f}{\partial \underline{x}}} + \frac{\partial g^T}{\partial \underline{x}} \underline{\lambda} = \underline{0} \tag{3.26}$$

$$\underline{\frac{\partial L}{\partial \underline{u}}} = \underline{\frac{\partial f}{\partial \underline{u}}} + \frac{\partial g^T}{\partial \underline{u}} \underline{\lambda} = \underline{0} \tag{3.27}$$

$$\underline{\frac{\partial L}{\partial \lambda}} = \underline{g}(\underline{x},\underline{u}) = \underline{0} \tag{3.28}$$

It is to be noted that $\frac{\partial g}{\partial \underline{x}}$ is the Jacobian matrix in a power flow solution by Newton's method [31] and the satisfaction of eqn.(3.28) is simply an ordinary power flow. Hence if a power flow program by Newton's method is available, $\underline{\lambda}$ can be readily obtained. Thus by solving the <u>three</u> nonlinear equations (3.26), (3.27) and (3.28) iteratively, the minimum can be obtained,

using eqn. (3.27) for updating the control variables. The iterative steps in the solution algorithm using the gradient method are as follows

1. Assume a set of contro parameters $\underline{u}$
2. Find a feasible power flow solution by Newton's method
3. Solve from (3.26) for $\underline{\lambda}$,

$$\underline{\lambda} = - \left[\frac{\partial g}{\partial \underline{x}} \right]^{T^{-1}} \frac{\partial f}{\partial \underline{x}} \qquad (3.29)$$

4. Insert $\underline{\lambda}$ from (3.29) into (3.27) and compute the gradient vector

$$\nabla \underline{f} = \frac{\partial L}{\partial \underline{u}} = \frac{\partial f}{\partial \underline{u}} + \left[\frac{\partial g}{\partial \underline{u}} \right]^{T} \underline{\lambda} \qquad (3.30)$$

5. If $\underline{\nabla f}$ is sufficiently small, the minimum has been reached.
6. Otherwise find a new set of control parameters from

$$\underline{u}^{new} = \underline{u}^{old} - c \ \underline{\nabla f} \qquad (3.31)$$

and return to step 2.

c in (3.31) is the step length in the gradient direction.

A careful observation of eqn.(3.27) reveals that it holds only if no inequality limit is imposed. Thus, at the optimum where some of the variables are on their limits, $\frac{\partial L}{\partial \underline{u}} \neq \underline{0}$. The handling of inequality limits for control variables offers no difficulty as they can

be set and held to their limits upon violation. When the dependent variables have violated their limits, the penalty method is used to force the variables back to their limits. The reason for using this approach are that these limits are not usually hard limits and that penalty terms can easily be incorporated.

This method offers a very neat and efficient algorithm for solution of the particular type of problem discussed with a careful choice of the step length c and the value of the weighting factor for penalty functions.

3.5 SUMMARY

A discussion of the solution techniques namely, the Method of Local Variations with suitable modifications, Augmented Penalty Function Method and the Optimal Power Flow Solution is provided in this chapter.

CHAPTER 4

SOLUTION TO THE LONG RANGE SCHEDULING PROBLEM

4.1 INTRODUCTION

In this chapter, the solution to the long range problem of optimal scheduling in hydro-thermal power systems is obtained using the Method of Local Variations and the Augmented Penalty Function Method, already discussed in Chapter 3. The mathematical model used here has been described in Sec. 2.2, where the transmission losses have been approximately considered using the loss coefficients and the limits on the operating voltage magnitudes, reactive powers and line flows etc. are not considered.

As already pointed out in Chapter 1, several attempts have been made in the past to solve the above problem using the method of Lagrange Multipliers, Incremental Dynamic Programming (IDP), Dynamic Programming, and the Pontryagin's maximum principle (continuous as well as discrete). The method of Lagrange multipliers used by Dandeno etal [8] does not consider the limits on the generations and hence, the solution obtained may not be practicable. Further the choice of the Lagrange multipliers and the γ-conversion factors (see Sec. 1.2) becomes very

difficult as the number of hydro stations increases. The formulation of the problem by Dynamic Programming suffers from the excessive computer storage requirements even for smaller systems. The formulation by the Pontryagin's maximum principle (both discrete as well as continuous versions) is essentially a control problem and becomes quite unwieldy for large systems. Further in all the indirect methods as this, the constraints on the state variables present considerable difficulty in handling them. Out of the various attempts made in the past, the IDP procedure adopted by Bernholtz and Graham [10] has been found to be most promising and hence a direct comparison of this method with the proposed method in this chapter is attempted in Sec. 4.6.

The solution to the above problem as given here essentially consists of a discrete time description of the system, a decomposition of the combined system into a hydro and thermal subsystems, a starting nominal trajectory of hydro reservoir storages and a systematic iterative method of perturbing the nominal trajectory such that the cost of thermal generations is monotone decreasing from iteration to iteration till a satisfactory convergence to an optimal trajectory is obtained. At every stage of the algorithm, all the constraints of the problem are satisfied. In each iteration, starting

with a nominal trajectory of reservoir storages, knowing the predicted load demand and inflow data, the discharge rates of the reservoirs during the subinterval as well as the corresponding hydro generations are computed. Having fixed the hydro generations over the entire interval of optimization, it remains to solve a sequence of mathematical programming problems (one for each subinterval) to obtain the corresponding thermal generations for each subinterval during the minimum cost of thermal generation. Thus this problem becomes a two level optimization problem and the details are discussed in subsequent sections.

For purposes of illustration (i) a one hydro and one thermal problem and (ii) a three hydro and four thermal problem have been considered. A detailed comparison of the MLV with IDP used by Bernholtz and Graham is provided and a suitable recommendation of the choice of the step size in the MLV algorithms is also made based on the computational experience.

4.2 FORMULATION OF A ONE-HYDRO-ONE-THERMAL PROBLEM

Consider an optimal scheduling problem of a one hydro and one thermal system. Let the total interval of optimization be divided into N equal subintervals (however in the method considered, the subintervals need not be of equal length). Let the hydro station state equation be

$$x(k+1) = x(k) + L(k) - u(k) - e(k), \quad (k=1,\ldots,N) \qquad (4.1)$$

x(1) and x(N+1) are specified from the requirement of the storages in the reservoirs at the beginning and end of the scheduling interval respectively. The performance index

$$J = \sum_{k=1}^{N} F(P_T(k)) \tag{4.2}$$

is to be minimized subject to the equality constraint

$$P_D(k) - P_H(k) - P_T(k) + P_{Loss}(k) = 0, \quad (k=1,\ldots,N) \tag{4.3}$$

and the inequality constraints

$$x_{min} \leq x(k) \leq x_{max}, \quad (k=2,\ldots,N) \tag{4.4}$$

$$u_{min} \leq u(k) \leq u_{max}, \quad (k=1,\ldots,N) \tag{4.5}$$

$$P_{Tmin} \leq P_T(k) \leq P_{Tmax}, \quad (k=1,\ldots,N) \tag{4.6}$$

where

$$F(P_T(k)) = aP_T(k) + bP_T^2(k) \tag{4.7}$$

where a and b are constants to be determined from the fuel cost-output characteristics of the thermal station.

$$P_{Loss}(k) = \sum_{i=1}^{2} \sum_{j=1}^{2} P_i(k)\, B_{ij} P_j(k), \quad (k=1,\ldots,N) \tag{4.8}$$

and

$$P_H(k) = \frac{H_o}{G}\left[1 + \frac{c}{2}(x(k) + x(k+1))\right] u(k), \quad (k=1,\ldots,N) \tag{4.9}$$

In eqn.(4.8), $P_1(k) \triangleq P_H(k)$ and $P_2(k) \triangleq P_T(k)$.

4.3 DESCRIPTION OF THE SOLUTION ALGORITHM

The algorithm starts with the choice of an initial state trajectory of reservoir storages, $x^1(k)$, $(k=1,\ldots,N+1)$. The choice of initial trajectory can be made keeping in view the physical considerations (note that the trajectory need not be a feasible one to start with). From the assumed nominal trajectory, the associated discharge rates and the corresponding hydro generations in each subinterval are calculated using eqns.(4.1) and (4.9) respectively. The constraints on the storages and the corresponding discharge rates are verified. Having determined $P_H(k)$, $(k=1,\ldots,N)$, eqn.(4.3) becomes in this case, a quadratic equation in $P_T(k)$ for each k. It is solved for each k to obtain the corresponding thermal generation $P_T(k)$. A choice has to be so made in this case from the two solutions of the quadratic equation that the upper and lower limits on the thermal generation are satisfied and the one which gives lesser cost for the subinterval under consideration. The subinterval costs as well as the total cost are computed and stored. If at any stage, the limits on either the reservoir storage, discharge rate, or the thermal generation are violated, the algorithm corrects the initial assumed trajectory by changing the unsatisfactory state variables suitably as described in Sec. 3.2 until a feasible trajectory is obtained.

Now the initial nominal trajectory (feasible) is varied iteratively using the following local variations procedure to obtain the optimal trajectory. Consider the variation of the nominal trajectory $x^j(k)$, $(k=1,\ldots,N+1)$ in the j-th iteration to obtain the nominal trajectory $x^{j+1}(k)$, $(k=1,\ldots,N+1)$ for the (j+1)-th iteration. Since the initial and final reservoir storages are specified, the local variations operation is to be performed at time instants $i=2,\ldots,N$. Let it be assumed that variation of the state value at the k-th instant is under consideration. Variation of the state values at time instants $i = 2,\ldots,k-1$ have already been performed, so that the current state trajectory at instants $2,\ldots,N$ is $\tilde{x}^j(i)$, $(i=2,\ldots,k-1)$, $x^j(i)$, $(i=k,\ldots,N)$. Here $\tilde{x}^j(i)$, $(i=2,\ldots,k-1)$ represents new values assigned to the nominal state trajectory in the j-th iteration, by the application of the MLV at time instants $i = 2,\ldots,k-1$. Now the value of the state at instant k is varied to $x^j(k) + \Delta x$, keeping all other state values unchanged. Tests are performed to see whether (1) this state value is admissible (2) the discharge rates required to take the state $\tilde{x}^j(k-1)$ to $x^j(k+1)$ via $x^j(k) + \Delta x$ are admissible and (3) the changed performance index value for the two subintervals (k-1) and k taken together is less than the corresponding performance index value for the same two subintervals

for the trajectory $\tilde{x}^j(k-1) \rightarrow x^j(k) \rightarrow x^j(k+1)$. If the answer is in the affirmative for the above three tests, then the state value at the k-th instant is changed to $\tilde{x}^j(k) = x^j(k) + \Delta x$. If the answer is in the negative, the same three tests are performed for $x^j(k) - \Delta x$. If this proves successful, the state value at the k-th instant is changed to $\tilde{x}^j(k) = x^j(k) - \Delta x$. Otherwise the old value is retained at the k-th instant and the local variations operation is performed at instant (k+1). In this way the Local Variations operation is applied successively upto the time instant N, to complete the iteration and obtain the nominal trajectory for the (j+1)-th iteration. These iterations which ensure a decrease in the values of the functional in each iteration are performed till a satisfactory convergence to an optimum has been obtained.

4.4 VARIATION OF THE STEP SIZE IN THE METHOD OF LOCAL VARIATIONS ALGORITHM

In the MLV algorithm discussed in Sec. 4.3, it is assumed that the step size Δx (i.e., the step size used in the variation of the hydro trajectory) is held constant throughout the iterative process. One would tend to feel that the choice of Δx has to be based on the engineering judgement taking into consideration the desired accuracy in the final schedule. However, during the course of this work

it was observed that, starting with a very large step size (Δx can be as large as the maximum allowable storage of the reservoir itself) and gradually reducing it towards the optimum resulted in a fast convergence in all the cases studied. The procedure using a variable step size is as follows. Convergence is obtained initially using a broad neighbourhood around the nominal trajectory, that is one corresponding to a large step size Δx and then considering the trajectory thus obtained as the nominal trajectory, the variation process is continued employing MLV to obtain another better approximation to the optimum trajectory with a smaller neighbourhood (say with a step size equal to half the previous value) and so on, until a satisfactory convergence to the optimum is obtained. This approach yielded the optimum trajectory with significantly less computational time than that corresponding to a small constant step size used throughout the iterative process.

4.5 NUMERICAL EXAMPLE

In this section, a numerical example which is similar to the one solved by Bernholtz and Graham is solved using the MLV algorithm described above for a one-hydro-one-thermal problem. The results will be presented for both constant step size and variable step size procedures as indicated above. The problem

considered infact is a short range scheduling problem of one day subdivided into 24 hourly subintervals. Bernholtz and Grahams' [10] formulation corresponds to the approximate model discussed in this chapter. The motivation for this is that a comparison of the MLV and the IDP procedures can be made (Sec. 4.6). The data as taken from Ref. [10] is given below.

Initial storage	=	711,000 ft^3/sec.-hour
Final storage	=	0.0 ft^3/sec.-hour
Maximum allowable storage	=	711,000 ft^3/sec.-hour
Minimum allowable storage	=	0.0 ft^3/sec.-hour
Maximum allowable discharge discharge	=	36,000 ft^3/sec.
Minimum allowable discharge	=	0.0 ft^3/sec.
Optimization interval considered	=	24 hours (one day) divided into 24 hourly subintervals

Fuel cost function used for the thermal station is

$\tilde{F}(P_T) = 373.704 + 9.60644P_T + 0.0019911P_T^2$ millions of BTu/hour. Cost of fuel is assumed to be 35 cents per million BTu.

$F(P_T) = 0.35\ \tilde{F}(P_T)$ dollars

Loss coefficients : $B_{11} = 0.00015$, $B_{22} = 0.00005$, $B_{12} = 0.00001$.

Maximum thermal generation	=	400 MW
Minimum thermal generation	=	0.0 MW
Maximum hydro generation	=	450 MW
Minimum hydro generation	=	0.0 MW
Basic head H_o	=	134.0 feet
c	=	2.0×10^{-7}
G	=	0.845×10^{-4}

Inflows into the reservoir and evaporation from the reservoir are not considered in this problem. Table 4.1 gives the initial nominal trajectory of the hydro reservoir storages. The optimal schedules obtained for this problem using the constant step size as well as variable step size procedures indicated in Secs. 4.3 and 4.4 respectively are illustrated here. Table 4.2 shows the optimal schedule of discharge rates, hydro and thermal generations with a constant step size of $\Delta x = 500.0$ ft^3/sec.-hour. Table 4.3 gives the optimal schedule of **discharge** rates, hydro and thermal generations using the variable step size procedure, starting step size being $\Delta x = 711{,}000.0$ ft^3/sec.-hour. Table 4.4 provides the optimal trajectories corresponding to constant step size and variable step size procedures. Computations are performed on IBM 7044 Computer at I.I.T. Kanpur, India.

TABLE 4.1: INITIAL NOMINAL TRAJECTORY OF HYDRO RESERVOIR STORAGES.

S.No.	Instant k (Beginning of the hour)	Initial storage in ft^3/sec.-hour
1	1	711,000
2	2	688,000
3	3	653,000
4	4	620,000
5	5	590,000
6	6	568,000
7	7	536,000
8	8	502,000
9	9	470,000
10	10	436,000
11	11	403,000
12	12	376,000
13	13	343,000
14	14	310,000
15	15	278,000
16	16	245,000
17	17	212,000
18	18	180,000
19	19	148,000
20	20	116,000
21	21	90,000
22	22	68,000
23	23	47,000
24	24	23,000
25	25 (end of 24th hour)	0

TABLE 4.2: OPTIMAL SCHEDULE FOR A ONE-HYDRO-ONE-THERMAL PROBLEM (for a constant step size Δx=500.0 ft^3/sec.-hour)

Hour	Load Demand in MW	Station generation in MW		Optimal discharge rate in ft^3/sec.
		Hydro	Thermal	
1	460	341.93	137.49	26,500
2	430	321.12	125.95	25,000
3	420	313.32	122.93	24,500
4	410	311.97	113.99	24,500
5	400	304.28	110.90	24,000
6	410	309.27	116.47	24,500
7	470	345.53	144.42	27,500
8	550	393.68	182.67	31,500
9	660	440.98	253.65	35,500
10	670	438.12	266.56	35,500
11	680	435.27	279.49	35,500
12	700	432.42	302.84	35,500
13	580	351.12	252.33	29,000
14	600	361.23	263.73	30,000
15	610	365.16	270.48	30,500
16	610	363.05	272.41	30,500
17	700	419.91	314.11	35,500
18	740	417.06	358.44	35,500
19	700	390.95	340.44	33,500
20	690	382.63	337.62	33,000
21	640	345.70	319.54	30,000
22	600	315.10	306.42	27,500
23	550	279.29	288.18	24,500
24	500	243.97	269.92	21,500

Initial cost in dollars = 23,727.0

Final cost in dollars = 20,399.6

TABLE 4.3: OPTIMAL SCHEDULE FOR A ONE-HYDRO-ONE-THERMAL PROBLEM (for a variable step size starting with Δx = 711,000 ft^3/sec.-hour)

Hour	Load Demand in MW	Station generation in MW		Optimal discharge rate in ft^3/sec.
		Hydro	Thermal	
1	460	355.17	125.43	27,528
2	430	328.90	118.82	25,612
3	420	315.38	121.04	24,668
4	410	302.35	122.86	23,751
5	400	287.71	126.23	22,694
6	410	298.81	126.14	23,668
7	470	331.33	157.42	26,362
8	550	382.76	192.55	30,611
9	660	439.85	254.65	35,389
10	670	441.80	263.27	35,778
11	680	433.45	281.12	35,332
12	700	436.03	299.58	35,777
13	580	357.69	246.30	29,528
14	600	368.73	256.86	30,612
15	610	370.25	265.81	30,917
16	610	368.09	267.79	30,917
17	700	419.65	314.35	35,471
18	740	416.80	358.68	35,472
19	700	389.72	341.56	33,389
20	690	382.05	338.15	32,943
21	640	349.59	315.94	30,332
22	600	320.22	301.65	27,943
23	550	281.51	286.08	24,694
24	500	245.23	268.72	21,612

Initial cost in dollars = 23,727.0

Final cost in dollars = 20,393.3

TABLE 4.4: OPTIMAL TRAJECTORIES OF RESERVOIR STORAGES

S.No.	Instant k (Beginning of the hour)	Optimal storage in ft^3/sec.-hour using a constant step size Δx=500.0 ft^3/sec.-hour	Optimal storage in ft^3/sec.-hour using a variable step size Δx=711,000 ft^3/sec.-hour
1	1	711,000	711,000
2	2	684,500	683,472
3	3	659,500	657,860
4	4	635,000	633,192
5	5	610,500	609,441
6	6	586,500	586,747
7	7	562,000	563,079
8	8	534,500	536,717
9	9	503,000	506,106
10	10	467,500	470,717
11	11	432,000	434,939
12	12	396,500	399,607
13	13	361,000	363,830
14	14	332,000	334,302
15	15	302,000	303,690
16	16	271,500	272,773
17	17	241,000	241,856
18	18	205,500	206,385
19	19	170,000	170,913
20	20	136,500	137,524
21	21	103,500	104,581
22	22	73,500	74,249
23	23	46,000	46,306
24	24	21,500	21,612
25	25 (end of 24th hour)	0	0

In addition to the results given for the constant and variable step sizes in Tables 4.2 and 4.3, two more trials with one constant and one variable step sizes are obtained and the computational time required for these are summarized below in Table 4.5.

TABLE 4.5: EFFECT OF STEP SIZE ON THE COMPUTATIONAL TIME (one-hydro-one-thermal problem)

Step size Δx in ft^3/sec.-hour	Cost in dollars		Computational time in secs. (Execution)
	Initial	Final	
711,000+	23,727.0	20,393.3	15.73
10,000+	23,727.0	20,397.0	17.77
500	23,727.0	20,399.6	20.45
250	23,727.0	20,380.6	45.76

Note: + in the above table indicates a variable step size.

4.6 COMPARISON OF THE INCREMENTAL DYNAMIC PROGRAMMING PROCEDURE WITH THE METHOD OF LOCAL VARIATIONS FOR A ONE-HYDRO-ONE-THERMAL PROBLEM

Bellman's dynamic programming algorithm proved to be inefficient for practical size problems, mainly because of the enormous computational requirements especially in terms of fast access memory in computers. Many modifications of the basic dynamic programming algorithm requiring significantly less storage have been developed in the past. Incremental Dynamic Programming (IDP) was one such method,

developed and successfully applied to the scheduling problem under consideration by Bernholtz and Graham [10]. It consists essentially of repetitively applying the dynamic programming algorithm locally, in a neighbourhood of a nominal trajectory. The word incremental refers to the fact that in each iteration, the search for an improved hydro schedule is confined to a neighbourhood of the nominal hydro schedule obtained in the preceding iteration.

The Method of Local Variations is yet another direct method employing search in a neighbourhood of a nominal trajectory as in Bernholtz and Grahams' method and having less computational requirements than the latter. Thus it can be seen that both of these methods involve direct search in a neighbourhood of a nominal trajectory. It is therefore natural to attempt a comparison of these methods from the aspect of computational requirements. In IDP, the best trajectory in a defined neighbourhood of a nominal trajectory is obtained by applying Bellman's principle of optimality. The resulting trajectory is made the nominal trajectory for the next iteration. This process is repeated to obtain convergence to a (locally) optimal trajectory. The MLV tries to obtain in each iteration only a better trajectory than the existing nominal trajectory in the neighbourhood under consideration.

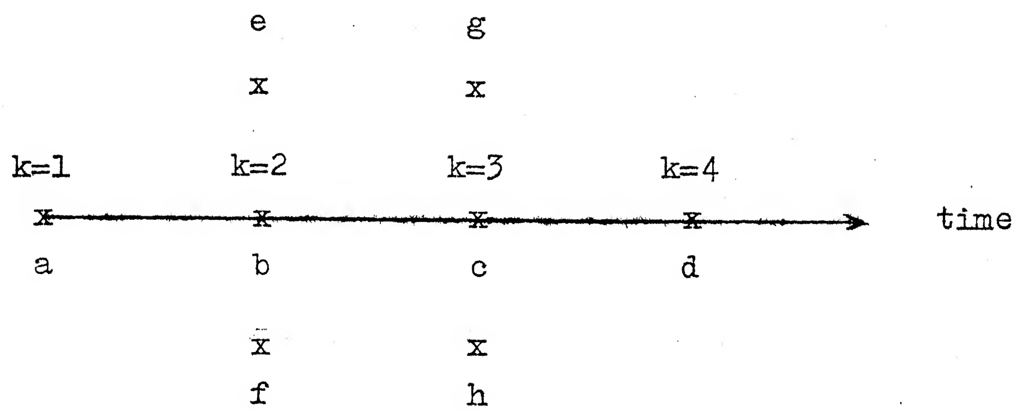

FIGURE 4.1: NOMINAL TRAJECTORY AND ITS NEIGHBOUHOOD UNDER CONSIDERATION (3 SUBINTERVAL CASE)

For the purpose of comparison of IDP and MLV methods for a one-hydro-one-thermal problem, the case in which the interval of optimization is subdivided into 3 equal sub-intervals is considered (see Figure 4.1). The points a, b, c, d represent the nominal state trajectory for a particular iteration number. The states represented by e, f, g, h indicate the neighbourhood considered in these methods. In the IDP method, 9 trajectories viz. aegd, aecd, aehd, abgd, abcd, abhd, afgd, afcd and afhd, need to be compared to find the optimal trajectory in this neighbour-hood assuming ofcourse that all these trajectories are feasible. In MLV, 5 trajectories are to be compared in the worst case, and 3 trajectories in the best case. This result is obtained as follows: For the worst case:
(1) abcd is compared with aecd resulting in failure, that is resulting in the answer that abcd is a better trajectory than aecd, (2) abcd is compared with afcd resulting in success (even if this results in failure the ultimate

result will be the same) (3) afcd is compared with afgd resulting in failure (4) afcd is compared with afhd resulting in success (or failure). Thus a maximum number of 5 trajectories need to be compared in the MLV. In a similar way it can be shown that in the best case only 3 trajectories need to be compared in this method.

In each iteration of IDP the neighbouring alternatives to the existing discharges are determined by measuring the so called 'weighted output' of the hydro system [10]. Thus in each iteration the quantized state values, the corresponding weights $G_1,\ldots,G_8$ and the corresponding discharges need to be determined and stored to locate the best trajectory in the neighbourhood. Thus for the three subinterval problem, 24 main storage locations are required for IDP. In MLV, it can be seen that 15 storage locations are required (4 for the nominal states a, b, c, d, 3 for the nominal discharges, 3 for the incremental performance index values in the three subintervals, 1 for the perturbed state, 2 for the two discharges in the two adjacent subintervals of the perturbed state, 2 for the incremental performance index values in these two subintervals corresponding to the perturbed state).

In this comparison, the number of calculations involved in either case is arrived at in the following way. In IDP while exploring all possible discharges which affect a transition from one stage to another stage, it is

observed that only 5 distinct discharges are involved and need to be considered. Each of these distinct discharges results in a different weighted output of the hydro system; thus each distinct discharge is associated with a single calculation. In each of the first and last subintervals, only two calculations are involved. Thus in all 9 calculations are involved per iteration in IDP method. In the worst case of MLV if the variation from point b to e results in failure, the variation from point b to f is tried. Each perturbation involves 2 calculations, corresponding to the discharges in the two adjacent subintervals. Thus 4 calculations are involved in the perturbation procedure referred to at the time instant k=2. A similar result holds for perturbations at the next time instant. Thus, in the worst case, 8 calculations are involved in MLV. Table 4.6 compares IDP and MLV for a one-hydro-one-thermal problem. It is assumed that identical procedures are used for calculations for the thermal subsystem in both cases.

From Table 4.6 it can be seen that the MLV requires significantly less number of storage locations, less number of trajectories to be investigated and fewer number of calculations than IDP.

The application of the MLV to higher dimensional problems, i.e. to problems of multireservoir hydrothermal power systems, a procedure similar to the

TABLE 4.6: COMPARISON OF THE MLV AND IDP PROCEDURES FOR A ONE-HYDRO-ONE-THERMAL PROBLEM

Number of sub-intervals	Incremental Dynamic Programming			Method of Local Variations				
	Trajectories to be compared	Primary storage required	No. of main calculations/iteration	Trajectories to be compared		Primary storage required	No. of calculations/iteration	
				Worst case	Best case		Worst case	Best case
3	9	24	9	5	3	15	8	4
6	243	51	24	11	6	24	20	10
N	3^{N-1}	3(3N-1)	(5N-6)	(2N-1)	N	3(N+2)	4(N-1)	2(N-1)

Successive Approximation to Dynamic Programming (S.A.D.P.) [32] is adopted as can be seen in Sec. 4.7. In this method, the state trajectory of one hydro reservoir is subjected to the MLV, keeping the trajectories of all the other hydro reservoirs at their nominal values determined in the previous iteration. After completing the variation of one trajectory over the interval of optimization, the variation of the trajectory of the second hydro reservoir is carried out in the same way. Thus the trajectories of all the hydro reservoirs are successively subjected to the MLV to complete one iteration. As it can be seen that an n-dimensional problem is split into n one-dimensional problems and hence the computational storage requirements, number of trajectories to be explored and number of calculations to be performed etc. will vary linearly with the number of the hydro stations under consideration. Similarly in the case of IDP, adopting the S.A.D.P. procedure makes the requirements vary linearly with the number of hydro stations. Hence the comparison given for one-hydro-one-thermal problem given in Table 4.6 is also applicable to the multidimensional case.

Thus it can be seen that for the same neighbourhood size, MLV is preferable to IDP, since the storage required and the calculations involved are less in the former case. When the initial trajectory is far away from the optimal one, MLV provides a faster transition from the current

nominal trajectory to the next one compared to IDP. If the optimal trajectory is encased in the neighbourhood at any stage of the iterative procedures of IDP and MLV, the IDP leads to the exact optimal trajectory, whereas MLV provides a trajectory which yields a reduction in the value of the performance index compared to the one corresponding to the existing trajectory. Thus it can be seen that it is advantageous to use MLV initially, till we are close to the optimal trajectory and then switch over to IDP for convergence to the optimal trajectory. Alternately the difference between the two optima yielded by IDP and MLV can be reduced by choosing smaller variational step size in MLV as the optimum is approached.

4.7 FORMULATION OF THREE-HYDRO-FOUR-THERMAL PROBLEM

In this section, the mathematical formulation is given for a power system consisting of three hydro and four thermal generating stations. The hydro stations are assumed to be of the storage reservoir type, which are independent of each other. The hydro system is described by the state equations

$$x_1(k+1) = x_1(k) + L_1(k) - u_1(k) - e_1(k) \qquad (4.11)$$

$$x_2(k+1) = x_2(k) + L_2(k) - u_2(k) - e_2(k) \qquad (4.12)$$

$$x_3(k+1) = x_3(k) + L_3(k) - u_3(k) - e_3(k) \qquad (4.13)$$

In eqns. (4.11) - (4.13), k varies from 1 to N and the initial and final state value $x_i(1)$ and $x_i(N+1)$, respectively $(i=1,2,3)$ are specified. The performance index

$$J = \sum_{k=1}^{N} \sum_{j=1}^{4} F_j(P_{Tj}(k)) \tag{4.14}$$

is to be minimized subject to the equality constraint

$$P_D(k) = \sum_{j=1}^{4} P_{Tj}(k) + \sum_{i=1}^{3} P_{Hi}(k) - P_{Loss}(k), \quad (k=1,\ldots,N) \tag{4.15}$$

and the following inequality constraints

$$P_{Tj\,min} \leq P_{Tj}(k) \leq P_{Tj\,max}, \quad (i=1,\ldots,4),(k=1,\ldots,N) \tag{4.16}$$

$$x_{i\,min} \leq x_i(k) \leq x_{i\,max}, \quad (i=1,\ldots,3),(k=2,\ldots,N) \tag{4.17}$$

$$u_{i\,min} \leq u_i(k) \leq u_{i\,max}, \quad (i=1,\ldots,3),(k=1,\ldots,N) \tag{4.18}$$

In eqn.(4.14)

$$F_j(P_{Tj}(k)) = a_j\, P_{Tj}(k) + b_j\, P_{Tj}^2(k) \tag{4.19}$$

Similarly in eqn. (4.15)

$$P_{Hi}(k) = \frac{H_{oi}}{G}[1 + \frac{c_i}{2}(x_i(k) + x_i(k+1))]u_i(k) \tag{4.20}$$

and $$P_{Loss}(k) = \sum_{i=1}^{7} \sum_{j=1}^{7} P_i(k)\, B_{ij}\, P_j(k) \tag{4.21}$$

In eqn.(4.21)

$$P_i(k) \triangleq P_{Hi}(k), \quad (i=1,\ldots,3)$$

$$P_i(k) \triangleq P_{Tj}(k), \quad (i = 4,\ldots,7), \ (j = 1,\ldots,4) \tag{4.22}$$

In the MLV, for the nominal trajectory at any stage of the iterative procedure, the corresponding discharge rates $u_i(k)$, and hydro generations $P_{Hi}(k)$, for all k can

be calculated from eqns. (4.11) to (4.13) and (4.20) respectively. To determine the corresponding optimal thermal generations, it is necessary to solve the following mathematical programming problem in each subinterval represented by k.

$$\text{Minimize} \sum_{j=1}^{4} F_j(P_{Tj}(k)) \tag{4.22}$$

subject to the equality constraint

$$P_D(k) = \sum_{j=1}^{4} P_{Tj}(k) + \sum_{i=1}^{3} P_{Hi}(k) - \sum_{i=1}^{7} \sum_{j=1}^{7} P_i(k) B_{ij} P_j(k) \tag{4.23}$$

and the inequality constraints

$$P_{Tj\ min} \leq P_{Tj}(k) \leq P_{Tj\ max}\ , \ (j=1,\ldots,4) \tag{4.24}$$

The Augmented Penalty Function Method, which has been discussed in detail in Chapter 3 has been chosen to solve the above problem.

4. 8 ALGORITHM DESCRIPTION

Step 1: Starting with a nominal trajectory for each hydro reservoir, the feasibility of the trajectory is established as described in Sec. 4.3. If the initial trajectory chosen is not a feasible one, then the algorithm automatically changes the trajectory to obtain a feasible one.

Step 2: The discharge rates and hydro generations are calculated in each subinterval for the nominal trajectory.

Step 3: With the hydro generations known over each of the subintervals, a mathematical programming problem stated in eqns. (4.22) to (4.24) is solved repeatedly for all the subintervals to determine the corresponding optimal thermal generations. The subinterval costs as well as the total cost over the entire interval of interest is also computed and stored.

Step 4: Taking one hydro station at a time (in some order), the initial trajectory is varied by a small increment Δx, in either direction using the MLV at time instant k=2. Feasibility of this variation is established as in step 1 above. Then the corresponding optimal thermal generations are determined in subintervals 1 and 2 by solving the mathematical programming problem for these two subintervals. The performance index value for these two subintervals is computed and compared with the corresponding value for the unvaried trajectory. If the variation considered results in a reduction in the performance index value, the variation is a success and the process is shifted to the instant k=3. Otherwise the variation $-\Delta x$ is tried. If neither of these variations results in a reduction in the cost functional for the two subintervals indicated, the variational process is shifted to the instant k=3, retaining the original value of the component at instant k=2. After the process is completed

at all instants k=2,...,N for the first hydro trajectory, the procedure is successively repeated with the second and third hydro trajectories at time instants k=2,...,N. It is to be noted here that while the variation process is applied to one hydro trajectory, the other hydro trajectories are held unchanged at their nominal values determined in the preceding iteration. This completes one iteration. Thus each iteration generates a feasible trajectory which is better than (or at least as good as) the trajectory in the previous iteration.

Step 5: The above iterations are continued until a satisfactory convergence to the optimum cost is attained. This is judged from the difference in performance index values for two consecutive iteration numbers; this difference must be less than a prespecified small value for satisfactory convergence.

4.9 EXAMPLE

Three-Hydro-Four-Thermal Problem:

Results will be presented for the constant as well as variable step size as in Sec. 4.5.

	Hydro Reservoir		
	I	II	III
Maximum storage (initial storage) in meter^3/sec.-month	80.0	135.0	80.0
Final storage at the end of one year in meter^3/sec.-month	80.0	135.0	80.0

	Hydro Reservoir I	II	III
Minimum storage in meter^3/sec.-month	0.0	0.0	0.0
Maximum allowable discharge rate in meter^3/sec.	73.0	75.0	80.0
Minimum allowable discharge rate in meter^3/sec.	3.334	4.353	3.543
Maximum hydro generation in MW	89.03	57.61	75.0
Minimum hydro generation in MW	0.0	0.0	0.0
Basic head in meters, H_{oi}	98.0	50.0	75.0
Constant c_i	0.004	0.002	0.004

Optimization period considered is one year of 12 monthly subintervals.

Constant $G = 100.0$

Transmission Loss Coefficients: $B_{11} = B_{22} = 0.0005$

$B_{33} = B_{44} = 0.0008$

$B_{55} = B_{66} = 0.0007$

$B_{77} = 0.0009$ and

$B_{ij} = 0.0$ $(i \neq j)$

Thermal Station Characteristics:

	Thermal Station I	II	III	IV
a_j in $/MW-hr.	2.5	2.6	2.8	2.0
b_j in $/MW^2 hr.	0.05	0.06	0.08	0.08
Maximum Thermal generation in MW	50.0	50.0	50.0	50.0
Minimum Thermal generation in MW	0.0	0.0	0.0	0.0

	Hydro Reservoir		
	I	II	III
Minimum storage in meter^3/ sec.-month	0.0	0.0	0.0
Maximum allowable discharge rate in meter^3/sec.	73.0	75.0	80.0
Minimum allowable discharge rate in meter^3/sec.	3.334	4.353	3.543
Maximum hydro generation in MW	89.03	57.61	75.0
Minimum hydro generation in MW	0.0	0.0	0.0
Basic head in meters, H_{oi}	98.0	50.0	75.0
Constant c_i	0.004	0.002	0.004

Optimization period considered is one year of 12 monthly subintervals.

Constant G = 100.0

Transmission Loss Coefficients: $B_{11} = B_{22} = 0.0005$

$B_{33} = B_{44} = 0.0008$

$B_{55} = B_{66} = 0.0007$

$B_{77} = 0.0009$ and

$B_{ij} = 0.0$ $(i \neq j)$

Thermal Station Characteristics:

	Thermal Station			
	I	II	III	IV
a_j in \$/MW-hr.	2.5	2.6	2.8	2.0
b_j in \$/MW^2 hr.	0.05	0.06	0.08	0.08
Maximum Thermal generation in MW	50.0	50.0	50.0	50.0
Minimum Thermal generation in MW	0.0	0.0	0.0	0.0

TABLE 4.7: INFLOW DATA FOR THE HYDRO RESERVOIRS

Month	1	2	3	4	5	6	7	8	9	10	11	12
Inflow into hydro reservoir I in $meter^3/sec$.	0.8	10.0	11.0	11.9	17.0	18.5	27.6	43.8	56.4	40.3	30.1	46.3
Inflow into hydro reservoir II in $meter^3/sec$.	23.0	17.0	12.0	10.0	20.0	40.0	53.0	63.0	62.0	48.0	55.0	48.0
Inflow into hydro reservoir III in $meter^3/sec$.	0.8	10.0	11.0	11.9	17.0	18.5	27.6	43.8	56.4	40.3	30.1	46.3

Table 4.7 shows the inflows into the reservoirs. Evaporations from the reservoirs are neglected in this problem.

CONSTANT STEP SIZE CASE: (Δx = 0.5 meter^3/sec.-month for all the hydro reservoirs).

Table 4.8 shows the initial and optimal trajectories for the three hydro reservoirs.

TABLE 4.8: INITIAL AND OPTIMAL TRAJECTORIES OF HYDRO RESERVOIR STORAGES

Instant k	Storage in the Hydro Reservoir in meter^3/sec.-mon					
	Hydro reservoir I		Hydro reservoir II		Hydro reservoir III	
	Initial	Optimal	Initial	Optimal	Initial	Optimal
1	80.0	80.0	135.0	135.0	80.0	80.0
2	75.0	73.0	128.0	128.0	75.0	75.0
3	70.0	70.5	123.0	123.0	70.0	69.5
4	65.0	64.5	120.0	119.5	65.0	65.0
5	60.0	67.5	117.0	104.5	60.0	66.5
6	58.0	65.5	114.0	99.0	58.0	64.0
7	53.0	61.0	110.0	103.0	53.0	59.0
8	59.0	59.0	115.0	87.5	59.0	57.0
9	68.0	70.0	119.0	89.5	68.0	69.0
10	72.0	80.0	122.0	135.0	72.0	80.0
11	77.0	80.0	128.0	135.0	77.0	80.0
12	78.0	78.0	131.0	131.0	78.0	78.0
13	80.0	80.0	135.0	135.0	80.0	80.0

Table 4.9 shows the optimal discharges for the hydro reservoi over the entire scheduling interval. Table 4.10 gives the

optimal schedules for hydro and thermal stations along wi the average load demand on the system. Tables 4.11 to 4. provide the optimal reservoir storage, discharge rates ar optimal schedule of hydro and thermal generations respectively using a variable step size procedure discussed in Sec. 4.4.

TABLE 4.9: OPTIMAL DISCHARGE RATES FOR HYDRO RESERVOIRS

Subinterval k (month)	Optimal discharge rate in meter^3/sec.		
	Hydro reservoir I	Hydro reservoir II	Hydro reservoir III
1	7.8	30.0	5.8
2	12.5	22.0	15.5
3	17.0	15.5	15.5
4	8.9	25.0	10.4
5	19.0	25.5	19.5
6	23.0	36.0	23.5
7	29.6	68.5	29.6
8	32.8	61.0	31.8
9	46.4	16.5	45.4
10	40.3	48.0	40.3
11	32.1	59.0	32.1
12	44.3	44.0	44.3

TABLE 4.10: OPTIMAL SCHEDULE FOR THE THREE-HYDRO-FOUR-THERMAL PROBLEM

Sub-interval k (month)	Station Generation in MW							Load demand in MW	Total generation in MW
	Hydro I	Hydro II	Hydro III	Thermal I	Thermal II	Thermal III	Thermal IV		
1	9.98	18.95	5.70	50.0	45.49	38.04	38.04	200	206.20
2	15.77	13.76	14.98	50.0	45.38	38.09	38.09	210	216.06
3	21.16	9.63	14.75	50.0	43.13	36.31	36.31	205	211.29
4	11.02	15.3	9.85	45.92	38.21	32.51	32.51	180	185.32
5	23.57	15.34	18.44	43.99	36.54	31.24	31.24	195	200.36
6	28.24	21.64	21.96	40.95	33.93	29.25	29.25	200	205.26
7	35.97	40.77	27.35	37.20	30.73	26.84	26.84	220	225.70
8	40.44	35.9	29.86	31.23	25.63	22.96	22.96	204	208.98
9	59.11	10.10	44.20	24.31	19.73	18.47	18.47	189	194.39
10	52.13	30.48	39.90	24.69	19.93	18.51	18.51	199	204.15
11	41.40	37.35	31.68	31.06	25.40	22.61	22.61	207	212.11
12	57.13	27.85	43.72	23.92	22.82	12.86	12.86	198	201.16

Initial cost in dollars = 7082.77

Final cost in dollars = 6991.77

VARIABLE STEP SIZE: (Δx = 80.0 meter^3/sec.-month for all the hydro reservoirs).

TABLE 4.11: INITIAL AND OPTIMAL TRAJECTORIES OF HYDRO RESERVOIR STORAGES

Instant k	Storage in the Hydro Reservoir in meter^3/sec.-month					
	Hydro reservoir I		Hydro reservoir II		Hydro reservoir III	
	Initial	Optimal	Initial	Optimal	Initial	Optimal
1	80.0	80.0	135.0	135.0	80.0	80.0
2	75.0	65.0	128.0	123.0	75.0	65.0
3	70.0	70.0	123.0	105.0	70.0	70.0
4	65.0	65.0	120.0	97.0	65.0	65.0
5	60.0	67.5	117.0	96.5	60.0	62.5
6	58.0	63.0	114.0	105.0	58.0	60.5
7	53.0	55.5	110.0	85.0	53.0	58.0
8	59.0	56.5	115.0	87.12	59.0	56.5
9	68.0	69.25	119.0	134.81	68.0	68.31
10	72.0	79.81	122.0	134.87	72.0	79.81
11	77.0	79.81	128.0	131.0	77.0	79.81
12	78.0	78.0	131.0	131.0	78.0	78.0
13	80.0	80.0	135.0	135.0	80.0	80.0

TABLE 4.12: OPTIMAL DISCHARGE RATES FOR HYDRO RESERVOIRS

Subinterval k (month)	Optimal discharge rate in meter^3/sec.		
	Hydro reservoir I	Hydro reservoir II	Hydro reservoir III
1	15.8	30.0	15.8
2	5.0	22.0	5.0
3	16.0	30.0	16.0
4	9.4	18.0	14.4
5	21.5	20.5	19.0
6	26.0	31.5	21.0
7	26.6	73.0	29.1
8	31.0	60.9	32.0
9	45.8	14.3	44.9
10	40.3	47.9	40.3
11	31.9	58.9	31.9
12	44.3	44.0	44.3

TABLE 4.13: OPTIMAL SCHEDULE FOR THE THREE-HYDRO-FOUR-THERMAL PROBLEM

Sub-interval k(month)	Station Generation in MW							Load demand in MW	Total generation in MW
	Hydro I	Hydro II	Hydro III	Thermal I	Thermal II	Thermal III	Thermal IV		
1	19.97	18.94	15.28	47.18	39.0	32.71	32.71	200	205.79
2	6.22	13.76	4.76	50.0	50.0	43.79	43.79	210	212.32
3	19.91	18.42	15.24	48.71	40.52	34.16	34.16	205	211.12
4	11.65	10.82	13.55	46.12	38.29	32.46	32.46	180	185.35
5	26.57	12.23	17.75	44.40	36.82	31.34	31.34	195	200.45
6	31.52	18.92	19.48	41.71	34.50	29.58	29.58	200	205.29
7	31.91	43.43	26.82	37.97	31.31	27.15	27.15	220	225.74
8	38.08	35.68	29.98	32.02	26.30	23.46	23.46	204	208.98
9	58.31	8.74	43.65	25.13	20.44	19.05	19.05	189	194.37
10	52.10	30.43	39.87	24.53	19.92	18.64	18.64	199	204.13
11	41.14	37.26	31.49	31.32	26.06	22.42	22.42	207	212.11
12	57.13	27.85	43.72	23.92	22.82	12.86	12.86	198	201.16

Initial cost in dollars = 7082.77

Final cost in dollars = 6976.97

Table 4.14 shows the effect of variable step size on the computational time.

TABLE 4.14: EFFECT OF THE STEP SIZE Δx ON THE COMPUTATIONAL TIME

Step size Δx in meter^3/sec.-month (for all the three hydro stations)	Cost in dollars		Computational time in seconds (Execution)
	Initial	Final	
80.0^+	7082.77	6976.97	163
20.0^+	7082.77	6976.97	165
1.0	7082.77	6992.73	203
0.5	7082.77	6991.77	208

Note: + in the above table indicates a variable step size.

4.10 DISCUSSION

Numerical results are presented in Tables 4.1 to 4.5 for a one-hydro-one-thermal system and in Tables 4.7 to 4.14 for a three-hydro-four-thermal system respectively. Results are presented for both the constant step size and variable step size separately in both the cases. It can be seen from the numerical results that the variable step size procedure resulted in a faster convergence as compared to the constant step size case. The initial value in the variable step size procedure can be as large

as possible (note that it can infact be equal to the value of the maximum allowable storage in the reservoir itself; however, in a multireservoir system, it implies a value equal to the maximum allowable storage in the smallest reservoir).

Tables 4.5 and 4.14 summarize the effect of the variable step size on the computational time consumed in both the problems respectively. The computational time is significantly less with variable step sizes compared to the constant step size. Thus on the basis of the experience gained, it is recommended that the initial step size in MLV procedure should be as large as possible and then progressively reduced as the optimum is approached.

4.11 CONCLUSIONS

In this chapter, the long range optimal power generation schedule in a hydro thermal power system is obtained using an approximate model discussed in Sec.2.2. Method of Local Variations has been employed to modify the hydro trajectory in each iteration, starting from an initially assumed trajectory to reach the optimum. Augmented Penalty Function Method has been used to solve the mathematical programming problem encountered in each subinterval for solving the optimal thermal generations.

The Method of Local Variations has been found to be attractive due to its simplicity, efficiency and ease

of implementation. Limits on the hydro reservoir, the hydro and thermal generations have been efficiently taken care of by this method. Furthermore, the method requires less storage and seems to require less computational time than those for the Incremental Dynamic Programming procedure adopted by Bernholtz and Graham to solve the same optimization problem. A recommendation based on computational experience is made regarding the choice of the step size while working with the MLV algorithm. For faster convergence, it is recommended that a step size as large as is consistent with the problem is selected to start with and gradually reduced as the optimum is approached. This provides a significant reduction in computational time compared to the use of constant step size throughout the process.

CHAPTER 5

SOLUTION TO THE SHORT RANGE SCHEDULING PROBLEM

5.1 INTRODUCTION

In Chapter 4, the problem of long range optimal scheduling in hydro-thermal power systems is solved using an approximate model, wherein the characteristics of the electrical network have been represented by the loss coefficients and the limits on the voltages, reactive powers, line flows etc. are not considered. However, in a short range scheduling problem when the data available would be more precise, in addition to hydro system dynamics one would like to represent the electrical network more rigorously using the power flow equations, and consider the various constraints on voltages, reactive powers, line flows etc. Such a sophisticated model has already been described in Sec. 2.3. Thus the attention in this chapter is focussed to the solution of the problem using this model. The head variations due to the reservoir dynamics have also been taken into account for the sake of generality.

To the author's knowledge, so far only two attempts [33], [20] have been made in the past to solve the short range scheduling problem both using A.C. power flow model for the electrical power network

as in Sec. 2.3. However the hydro system dynamics has not been included in [33]. Both the above attempts are briefly summarized below. Ramamoorty and Gopala Rao [33] have formulated this problem as an additively separable nonlinear programming problem. The representation of the hydro system is as follows. It is assumed that the total energy available at each hydro station is known for the total interval of optimization. Under the assumption of constant head and equal length of each subinterval, it was possible to write a coupling constraint for the total interval of optimization for each hydro station, which stipulates that the available energy at each hydro station is utilized in power generation. Associating a dual variable (which has the physical significance of the equivalent water cost) with the coupling constraint at each hydro station, the problem thus reduces to minimizing an objective function (being the sum of the cost of thermal generation in all the subintervals), coupling equations defined by the total energy constraints as seen above, and the constraint sets defined by the load demand and the various operating constraints on the system in the corresponding subintervals. With a choice of the above mentioned dual variables the problem of N subintervals was split into N subproblems, one for each subinterval (which can be solved separately) using Lasdon's [22] decomposition technique. The solution

procedure consists of solving these subproblems iteratively using SUMT [35], while updating the dual variables in each iteration by a gradient technique until the convergence to the optimum solution is obtained.

As can be seen from the above discussion, no attempt was made by them to represent the dynamics of the hydro subsystem in their formulation. Thus there was no method of checking the operational constraints of the hydro subsystem, such as the upper and lower limits on the hydro reservoir storages and the associated discharge rates etc. Also the relaxation of the assumptions of constant head and equal length of each subinterval will make it very difficult to write the above mentioned coupling constraints for the hydro stations. Furthermore, unless the initial choice of the dual variables is proper, convergence to the optimum may be difficult in addition to the large computation time required. In the author's opinion, the hydro subsystem was not represented adequately and their method of solution was the same as that of solving the optimal scheduling problem of a purely thermal system except that the additional term in the cost function representing the equivalent cost of hydro power.

Bonaert, El-Abiad and Koivo [20] have solved the short range scheduling problem using a model similar to the one considered in this chapter. Their approach was basically the same as in [10]. The solution procedure in each iteration consists of essentially starting with a nominal trajectory of reservoir storages and applying the Dynamic Programming algorithm locally in a neighbourhood around the nominal trajectory and in the process to obtain the optimum trajectory. In each subinterval, knowing the hydro generations from the hydro subsystem, the corresponding optimal thermal generations are obtained by solving an optimal power flow problem. At the end of each iteration the existing nominal trajectory is replaced by the best trajectory in its neighbourhood. The difficulty with this method as seen from the numerical results reported [20] was that the computational storage requirements were significantly high even for the 5 bus system considered by them.

In this chapter, the solution to the above problem is obtained employing a similar decomposition of the combined system into hydro and thermal subsystems as that of Bonaert etal. The major difference between these two approaches stems from the method of modifying the initially chosen nominal trajectory in successive iterations. In the present method the MLV which has been applied in the long range scheduling problem in

Chapter 4 is also used for the short range scheduling problem. In each iteration, starting with a nominal trajectory of reservoir storages, the method consists of perturbing the initial trajectory at one instant at a time and choosing the best trajectory in two adjacent subintervals comparing the three possible alternatives. During the perturbation at any instant, the rest of the trajectory is assumed to remain fixed at its previous value. The perturbation process is performed at all instants, to complete one iteration. As in [20], knowing the hydro generations from the hydro subsystem in each subinterval, starting from the end of first subinterval to the beginning of the last subinterval, the corresponding optimal thermal generations are calculated. In Sec. 4.6, it has been shown that the storage requirements for the MLV are significantly less than those for the IDP. Thus it can be seen that the present method affords a better approach than that of Bonaert etal from the aspect of computer storage requirements. It can also be seen from the numerical results reported later in this chapter, that the computational time requirement is also less compared to that of Bonaert etal.

5.2 PROBLEM DEFINITION

The system configuration considered for short range scheduling problem in this chapter is as shown in Figure 5.1. This is similar to that discussed by

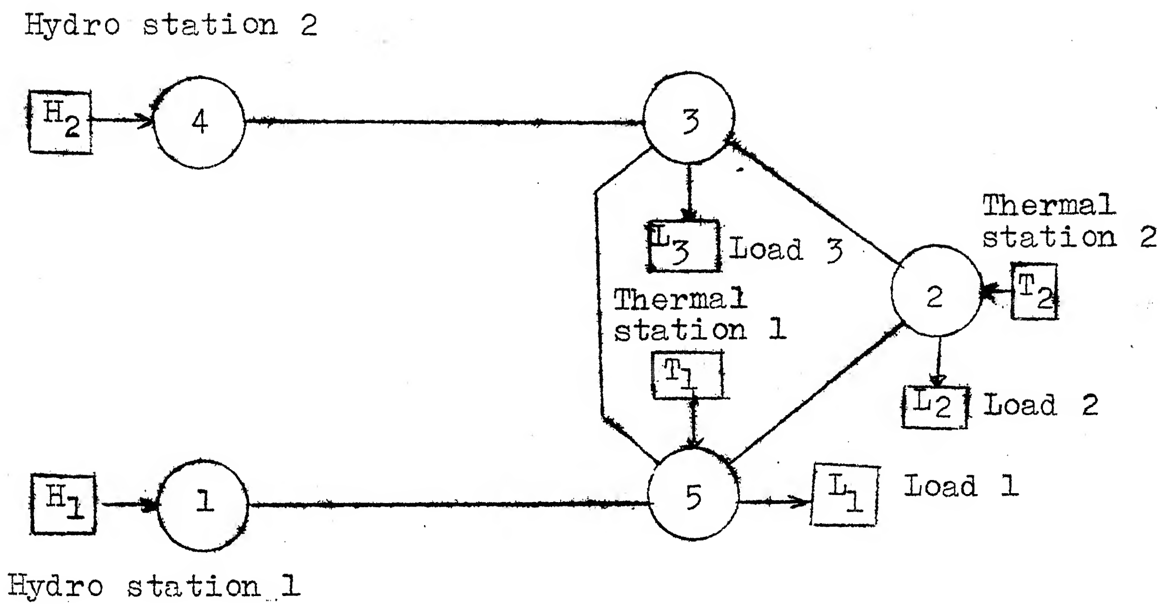

FIGURE 5.1: SYSTEM STUDIED FOR THE SHORT RANGE PROBLEM.

Bonaert etal [20] and the motivation for this is so that a comparison between the present approach and that of Bonaert etal can be made. The generation at buses 1 and 4 are from hydro stations, which are assumed to be operating independent of each other and thermal generations are assumed at buses 2 and 5. Loads L_1, L_2 and L_3 are assumed to be at buses 5, 2 and 3 respectively. The detailed mathematical model for a general short range problem has been described in Sec. 2.3. Let the total interval of optimization (one day) be divided into 6 equal subintervals of 4 hours each duration. It is assumed here that the

loads in each subinterval remain constant at all load buses. The initial and final storages of the reservoirs are specified.

The state equations describing the hydro subsystem dynamics are

$$x_i(k+1) = x_i(k) + L_i(k) - u_i(k), \ (i=1,2), \ (k=1,\ldots,6) \tag{5.1}$$

$x_i(1)$ and $x_i(6)$, $(i=1,2)$ are specified. The hydro generation at the i-th station during the k-th subinterval can be related to the water head and discharge rate by the relation [16]

$$P_{Hi}(k) = \frac{H_{oi}}{G}[1 + \frac{c_i}{2}(x_i(k) + x_i(k+1))]u_i(k) \ , \quad (i=1,2), \ (k=1,\ldots,6) \tag{5.2}$$

Bounds on the reservoir storages and discharge rates are

$$u_{i\ min} \leq u_i(k) \leq u_{i\ max} \ , \ (i=1,2), \ (k=1,\ldots,6) \tag{5.3}$$

$$x_{i\ min} \leq x_i(k) \leq x_{i\ max} \ , \ (i = 1,2), \ (k=2,\ldots,6) \tag{5.4}$$

The thermal subsystem can be formulated as follows (from Sec. 2.3). In each subinterval, having fixed the hydro generations at the two hydro stations from the nominal trajectory, it remains to solve an optimal power flow for each subinterval. Hence this problem becomes a static optimization problem in each subinterval which can be stated as

$$\text{Minimize } J = \sum_{k=1}^{6} \sum_{i=1}^{2} F_i(P_{Ti}(k)) \tag{5.5}$$

where J represents the total thermal generation cost subject to the following equality and inequality constraints.

Equality Constraints:

The real power balance is given by

$$I_i(k) - P_i(k) + C_i(k) = 0 \;,\; (i=1,2),\; (k=1,\ldots,6) \tag{5.6}$$

The reactive power balance is given by

$$K_i(k) - Q_i(k) + D_i(k) = 0 \;,\; (i=1,2),\; (k=1,\ldots,6) \tag{5.7}$$

where the real and reactive power injections at the i-th bus are respectively

$$I_i(k) = \sum_{\substack{j=1 \\ j\neq i}}^{5} V_i(k)V_j(k)Y_{ij}\text{Cos}(\theta_{ij}+\delta_i(k)-\delta_j(k)) + V_i^2(k)Y_{ii}\text{Cos}\,\theta_{ii} \;,\; (k=1,\ldots,6) \tag{5.8}$$

and

$$K_i(k) = \sum_{\substack{j=1 \\ j\neq i}}^{5} V_i(k)V_j(k)Y_{ij}\text{Sin}(\theta_{ij}+\delta_i(k)-\delta_j(k)) + V_i^2(k)Y_{ii}\text{Sin}\,\theta_{ii} \;,\; (k=1,\ldots,6) \tag{5.9}$$

Inequality Constraints:

The various operating restrictions in the system impose the following limits on the variables $P_i(k)$, $Q_i(k)$ and $V_i(k)$.

$$P_i^2(k) + Q_i^2(k) - S_{i\ max}^2 \leq 0 \tag{5.10}$$

$$P_{i\ min} - P_i(k) \leq 0 \tag{5.11}$$

$$Q_{i\ min} - Q_i(k) \leq 0 \tag{5.12}$$

$$Q_i(k) - Q_{i\ max} \leq 0 \tag{5.13}$$

$$V_{i\ min} - V_i(k) \leq 0 \tag{5.14}$$

$$V_i(k) - V_{i\ max} \leq 0 \tag{5.15}$$

The restrictions on the individual line flows to ensure system stability can be represented by

$$|\delta_i(k) - \delta_j(k)| \leq |T_{ij\ max}| \tag{5.16}$$

Eqns. (5.10) to (5.16) are to be satisfied in each subinterval. Thus solving the thermal subsystem for the optimal thermal generations for known hydro generations consists of minimizing (5.5) subject to equality constraints (5.6) and (5.7) and the upper and lower limits on the problem variables given by (5.10) to (5.16).

5.3 TWO LEVEL SOLUTION APPROACH

The combined system is decomposed into a dynamic hydro subsystem and a thermal subsystem (static in each subinterval). In the hydro subsystem, starting with a nominal (feasible) trajectory of reservoir storages, the nominal hydro generations in the various subintervals are obtained. Considering these hydro powers as injections into the electrical network at the appropriate buses, the thermal subsystem

in each subinterval is optimized using the Optimal Power Flow Solution due to Dommel and Tinney discussed in Sec. 3.4. The existing nominal hydro trajectory is now varied using the MLV iteratively to result in a reduction in the cost of thermal generation in each iteration, while an optimal power flow is obtained with the given hydro injections, for the thermal subsystem, in each subinterval. As can be seen, this solution procedure constitutes a two level approach, to coordinate the hydro power iteratively to result in an optimal schedule for the combined system, while the two subsystems are decoupled at the first level with the hydro power injections as the interaction variables. Figure 5.2 shows the flow chart of the algorithm, which is described in detail in the next section.

5.4 ALGORITHM DESCRIPTION

Step 1: To start with an initial nominal trajectory $x_i(k)$, $(k=1,\ldots,6)$, for all hydro stations, $(i=1,2)$ is chosen. Feasibility of the trajectory is established by making sure that the trajectory does not violate the constraints on states $x_i(k)$, and the resulting discharges $u_i(k)$, for all i and k. If the initial trajectory chosen is infeasible, the program changes the trajectory to obtain a feasible one as described in Sec. 3.2. Knowing the inflow data for all the reservoirs over the interval of optimization, the subinterval nominal discharge rates

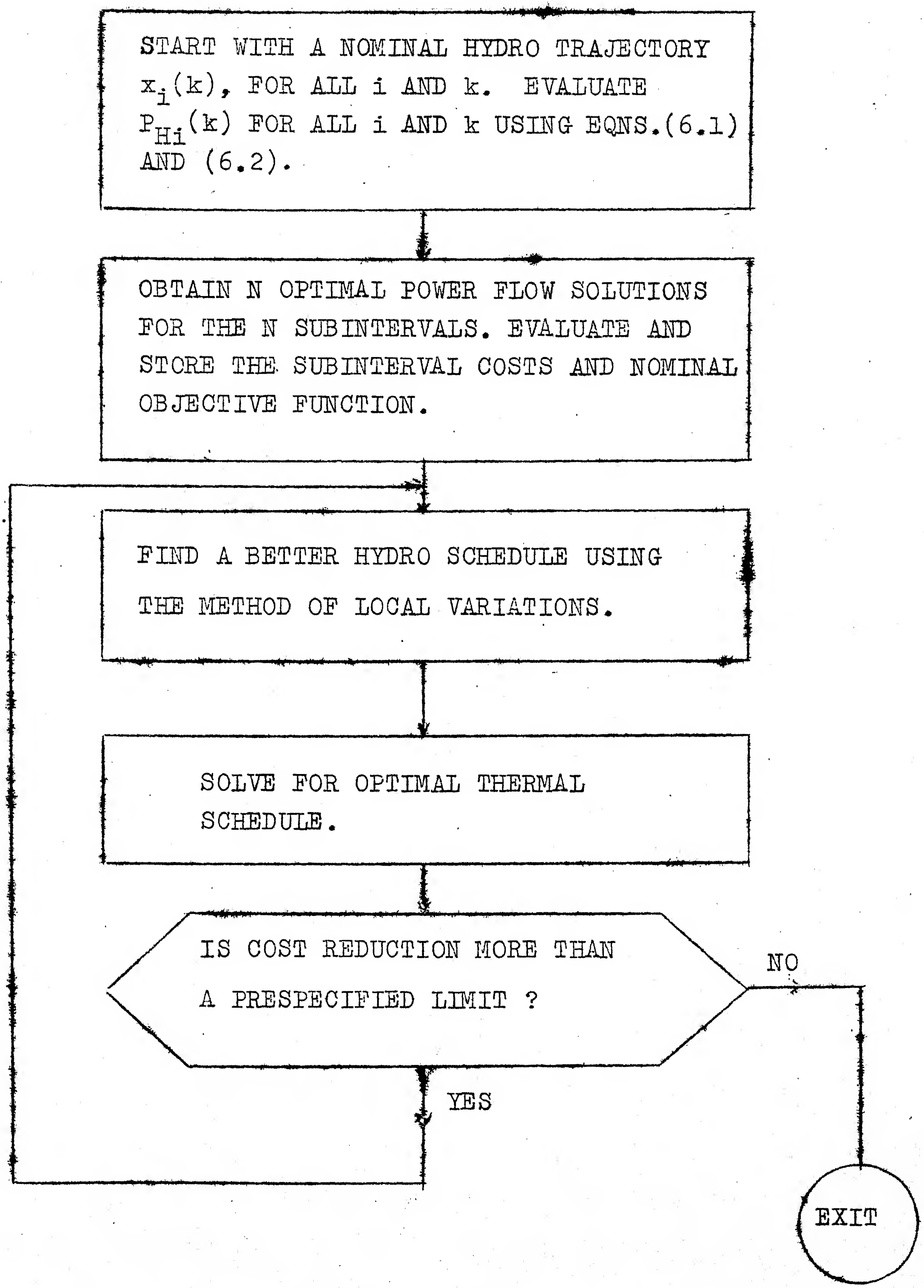

FIGURE 5.2: OPTIMAL SCHEDULING ALGORITHM FOR THE COMBINED SYSTEM.

unchanged, while the trajectory of one of the hydro station is varied. Feasibility of this variation is established as in step 1 above. The discharge rates and the hydro generations for subintervals 1 and 2 are obtained using eqns. (5.1) and (5.2) and the corresponding optimal thermal generations are determined by obtaining the optimal power flow solutions for these two subintervals. The subinterval costs for these two subintervals are obtained and compared with the corresponding two subinterval costs for the unvaried trajectory. If the variation considered results in a reduction in the cost functional for the two subintervals indicated, the variation is a success and the process is shifted to the instant k = 3. Otherwise the variation - Δx is tried. If neither of these variations results in a reduction in the cost functional for the two subintervals considered, the nominal trajectory value is retained at this time instant and the procedure is repeated at time instants k = 3,4,... etc. When this procedure has been applied to the states at all the time instants k = 2,...,6, the procedure is repeated with the second hydro station. When both the hydro stations have been tried, one iteration is said to have been completed.

Step 4: The above iterations are continued until a satisfactory convergence to the optimum cost is attained.

5.5 NUMERICAL EXAMPLE

The algorithm described in the above section uses throughout a constant step size vector for modifying the hydro trajectory in each iteration. Since, it has already been established in Chapter 4, that starting with as large a step size vector as possible initially and then gradually reducing it during the iterative process as the optimum is approached, results in a significant reduction in the computational time, in this section, numerical results obtained by variable step size vector alone are reported.

The data as taken from [20] is as follows. The system considered is composed of the standard five bus system proposed by IEEE (Figure 5.1). The line data and the load data are given in Tables 5.1 and 5.2 respectively. The data pertaining to the hydro and thermal systems are given in Tables 5.3 and 5.4. Each subinterval considered is of 4 hours duration. Table 5.5 gives the upper and lower limits on the reservoir storages at the various instants over the optimization interval. Table 5.6 stipulates the limits on the voltages, active and reactive powers at the various buses. Table 5.7 gives the initial and optimal trajectories of hydro reservoir storages. Table 5.8 gives the optimal schedule for hydro and thermal stations obtained using an initial step size

of Δx = 160.0 meter^3/sec.-4 hours for both the hydro stations and gradually reducing it as described in Sec. 4.4. Base power is taken to be 100 MVA. Bus number 5 is taken as the slack bus.

TABLE 5.1: LINE DATA (P.U.)

Line	1-5	2-5	3-5	2-3	3-4
Resistance	0.03	0.08	0.105	0.033	0.106
Reactance	0.103	0.262	0.347	0.118	0.403

TABLE 5.2: ACTIVE LOAD DATA (P.U.)

Sub-interval	1	2	3	4	5	6
Load L_1	0.6	0.8	1.0	0.9	0.7	0.5
Load L_2	0.3	0.4	0.5	0.45	0.35	0.25
Load L_3	0.3	0.4	0.5	0.45	0.35	0.25

Note: Reactive loads are 10 percent of active loads.

TABLE 5.3: CHARACTERISTICS OF THE HYDRO SUBSYSTEM

Hydro system	Hydro 1 (i=1)	Hydro 2 (i=2)
Initial storage in meter^3/sec.-4 hours	2330	2330
Final storage in meter^3/sec.-4 hours	2330	2330
Maximum allowable discharge in meter^3/sec.	500	500
Minimum allowable discharge in meter^3/sec.	100	100
Initial head of the reservoir in meters	14	14
Final head of the reservoir in meters	14	14
Basic head H_{oi}	10	10
Inflow (assumed constant throughout in meter^3/sec.	250	300

TABLE 5.4: THERMAL SUBSYSTEM DATA (Same for both stations)

Lower limit of power (p.u.)	0.1
Upper limit of power (p.u.)	1.0
Linear coefficient of thermal cost (\$/MW-hr.)	3.8
Quadratic coefficient of thermal cost (\$/MW^2-hr.)	0.01

TABLE 5.5: UPPER AND LOWER LIMITS ON THE RESERVOIR STORAGES

Instant k	1	2	3	4	5	6	7
$x_{2\ max}$ in meter^3/sec.-4 hrs.	2330	2365	2400	2430	2400	2365	2330
$x_{2\ min}$ in meter^3/sec.-4 hrs.	2330	2300	2260	2230	2260	2300	2330
$x_{1\ max}$ in meter^3/sec.-4 hrs.	2330	2400	2460	2500	2450	2390	2330
$x_{1\ min}$ in meter^3/sec.-4 hrs.	2330	2275	2218	2158	2200	2265	2330

TABLE 5.6: LIMITS ON VOLTAGES AND GENERATIONS

Bus number	VOLTAGE		REAL POWER		REACTIVE POWER	
	Upper limit (in p.u.)	Lower limit (in p.u.)	Upper limit (in p.u.)	Lower limit (in p.u.)	Upper limit (in p.u.)	Lower limit (in p.u.)
1^+	1.02	1.02	1.0	0.1	0.60	-0.60
2^+	1.04	1.04	1.0	0.1	0.60	-0.60
3	1.10	0.9	0.0	0.0	0.0	0.0
4^+	1.02	1.02	1.0	0.1	0.60	-0.60
5^+	1.04	1.04	1.0	0.1	0.60	-0.60

Note: + in the above table indicates voltage controlled bus.

TABLE 5.7: INITIAL AND OPTIMAL TRAJECTORIES OF THE HYDRO RESERVOIR STORAGES

Instant k	1	2	3	4	5	6	7
Initial storage $x_1(k)$	2330	2328	2328	2328	2328	2328	2330
Optimal storage $x_1^*(k)$	2330	2398	2458	2498	2448	2388	2330
Initial storage $x_2(k)$	2330	2328	2328	2328	2328	2328	2330
Optimal storage $x_2^*(k)$	2330	2363	2398	2423	2398	2363	2330

Note: In the above table the units of the storages are same as defined earlier.

TABLE 5.8: OPTIMAL SCHEDULE FOR HYDRO AND THERMAL STATIONS

Sub-interval k	1	2	3	4	5	6
$P_{H1}(k)$	0.2207	0.2315	0.2568	0.3667	0.3775	0.3734
$P_{H2}(k)$	0.3234	0.3218	0.3347	0.3955	0.4068	0.4033
$P_{T1}(k)$	0.4208	0.6819	0.8654	0.6114	0.3408	0.1485
$P_{T2}(k)$	0.2503	0.3806	0.5606	0.4503	0.3000	0.1001
$u_1(k)$	182.0	190.0	210.0	300.0	310.0	308.0
$u_2(k)$	267.0	265.0	275.0	325.0	335.0	333.0

Note: All units are as defined before except $u_1(k)$ and $u_2(k)$ which are expressed in meter^3/sec.

Table 5.9 gives the voltages, phase angles and reactive powers at all the buses over the entire interval of optimization (for the optimum condition).

TABLE 5.9: VOLTAGES, PHASE ANGLES AND REACTIVE POWERS AT THE VARIOUS BUSES

Sub-interval k	1	2	3	4	5	6
V_1	1.02	1.02	1.02	1.02	1.02	1.02
δ_1	1.679	1.744	1.899	2.567	2.632	2.607
V_2	1.04	1.04	1.04	1.04	1.04	1.04
δ_2	-0.368	-0.831	-0.690	-0.493	-0.167	-0.534
V_3	1.0271	1.0243	1.0214	1.0223	1.0249	1.0277
δ_3	0.029	-0.80	-1.111	-0.422	0.354	0.485
V_4	1.02	1.02	1.02	1.02	1.02	1.02
δ_4	7.862	7.001	6.972	9.034	10.042	9.832
V_5	1.04	1.04	1.04	1.04	1.04	1.04
δ_5	0.0	0.0	0.0	0.0	0.0	0.0
Q_1	-0.2581	-0.2609	-0.2674	-0.2949	-0.2975	-0.2965
Q_2	0.1587	0.1854	0.1983	0.2020	0.1827	0.1777
Q_3	0.0	0.0	0.0	0.0	0.0	0.0
Q_4	-0.0808	-0.0738	-0.0682	-0.0768	-0.0842	-0.0897
Q_5	0.3584	0.3689	0.4033	0.4400	0.4338	0.3998

Note: $Q_1,\ldots,Q_5$ are the reactive powers at buses $1,\ldots,5$ respectively. All units are in p.u. except the phase angles (δ's), which are given in degrees.

Initial cost in dollars = 7805.76

Final cost in dollars = 7779.36

Table 5.10 shows the effect of variable step size on the computational time. In addition to the variable step size case presented in detail, three more cases (one variable step size and two constant step sizes) are summarized in this table. Table 5.11 draws a comparison between the proposed method and that of Bonaert etal from the aspect of core storage and computational time requirements.

TABLE 5.10: EFFECT OF VARIABLE STEP SIZE ON COMPUTATIONAL TIME

Step size Δx in meter^3/sec.-4 hrs. (for both the hydro reservoirs)	Cost in dollars		Computational time in secs. (Execution)
	Initial	Final	
160^+	7805.76	7779.36	177
40^+	7805.76	7779.36	177
10	7805.76	7779.88	265
5	7805.76	7779.36	513

Note: $^+$indicates a variable step size.

TABLE 5.11: COMPARISON OF THE COMPUTATIONAL TIME AND STORAGE REQUIREMENTS FOR THE PROPOSED METHOD WITH THAT OF BONAERT ETAL.

Method	Core storage required in word memory	Computational time in secs. (Execution)
Bonaert etals' method (CDC 6500 computer)	47.2K	200
Proposed method (IBM 7044 computer)	17.29K	177

5.6 DISCUSSION

As seen earlier in this chapter, both the present approach and that of Bonaert etal [20] use the Optimal Power Flow Method in solving the thermal subsystem for optimal thermal generations. Though the actual steps in the solution are different in both the approaches, it can be seen easily that the computer storage required for thermal subsystem in either case remains same. The storage primarily consists of the subinterval costs, total cost and the subinterval optimal thermal generations etc.

But in the hydro subsystem Bonaert etal have used the IDP and the MLV is used in this approach. As already seen, the basic advantage of employing the MLV approach is for the hydro subsystem calculations. In Sec. 4.6, as also in [36] both the MLV and IDP algorithms have been directly compared from the aspects of computational requirements and it was established that in modifying the hydro trajectory in each iteration, the MLV requires remarkably less computational storage compared to the IDP procedure. This in the author's opinion is the main advantage of the present approach over that of Bonaert etal. The computer storage and time requirements of both the methods are given in Table 5.11. It can be easily seen from this table that the computer storage requirements for this method are less than half that of Bonaert etal. Regarding the computer time, the calculations are carried on CDC 6500 computer by Bonaert etal, whereas IBM 7044 computer was used in this case, which is a much slower machine. The time required to obtain the solution to the numerical problem in Sec. 5.5 is 177 secs. on IBM 7044 whereas on CDC 6500, the time of computation as given in [20] was 200 secs. This clearly shows that there is a significant saving in computational time also, with the present method. Both the saving in storage and computation time make this method more attractive especially for a large interconnected power system.

5.7 CONCLUSIONS

In this chapter, the short range optimal scheduling problem using a sophisticated model for a hydro thermal power system has been solved using the Method of Local Variations and Optimal Power Flow Method. The combined system has been decomposed into a hydro subsystem and a thermal subsystem and a two level solution approach is employed. The hydro subsystem has been represented by its dynamics considering the deterministically known inflows into the reservoirs and the load demand. The thermal subsystem has been represented by the A.C. power flow model. This would enable the limits on the voltage magnitudes, reactive powers and the line flows etc. in the electrical network to be considered. The modification of the hydro trajectory is achieved in each iteration using the MLV, while the optimal thermal generations are obtained in each subinterval by the Optimal Power Flow Method.

Results are presented for a 5 bus system with two hydro and two thermal stations. A comparative discussion between this method and that of Bonaert etal is presented. The requirements of core storage and computation time for this method have been observed to be significantly less than those of Bonaert etal for the same 5 bus system. Hence the proposed method would be more attractive for large interconnected power systems.

CHAPTER 6

OPTIMAL SCHEDULING OF PUMPED STORAGE PLANTS

6.1 INTRODUCTION

In the problems considered in Chapters 4 and 5, it has been implicitly assumed that the hydro plants are operated purely as generating stations and no pumping operation has been considered at these stations. The main function of a pumping cum generating (pumped storage) plant is to pump water from the back-bay reservoir to the fore-bay reservoir during the periods of low demand and use the water thus stored for generation purposes during the periods of high demand. To utilize the capacity of a pumped storage plant in a combined hydro-thermal power system in the most efficient manner, it is necessary to perform an optimization study for the system, aimed at the determination of an optimal pumping and generating schedule for the pumped storage plant. The schedule should be such that the total fuel cost incurred in thermal generation during a specified optimization interval is minimum. The resulting schedule should ofcourse conform to the system operating constraints such as the limits on the storages of the reservoirs, discharge rates (pumping as well as generating), the thermal generations, the voltage magnitudes at various buses, the reactive powers and the line flows etc. as already discussed in Chapters 2 and 5.

The problem of pumped storage optimal scheduling is different from the purely hydro generation scheduling problem due to the fact that in a pumped storage plant, the plant operates as a generating station in certain sub-intervals and as a pumping station consuming power in the remaining subintervals of the optimization interval. Various attempts [37] to [41] have been made in the past to study scheduling with pumped storage plants, however most of them have dealt with specific cases rather than problem of a general nature discussed in this chapter. Bernholtz etal [37] have given a method using the modified dynamic programming procedure [42], [43] for the optimum scheduling of Ontario Hydro's Sir Adam Beck-Niagara generating station. For the problem considered, water from upper Niagara river enters the Beck fore-bay which is connected by a pumping cum generating station to the storage reservoir and to the tailrace by means of a generating station respectively. The discharge cum pumping rates of the pumping cum generating station as well as the discharge rates of the Beck generating station depend on the volumes of water available in both the storage reservoir and the Beck fore-bay. The discrete scheduling problem consists of determining N pairs of discharges through pumping cum generating and the generating stations over the N subintervals of optimization leading from a specified initial state (E_{F1}, E_{R1})

to a specified final state ($E_{F(N+1)}$, $E_{R(N+1)}$) where E_F and E_R refer to the volumes of the water in the fore-bay and the storage reservoirs respectively, while maximizing a suitably defined objective function satisfying the various constraints on storages and discharge rates etc. The solution procedure consists of applying the Dynamic Programming starting backwards from the last subinterval. At the beginning of each subinterval, a two way table is constructed with both the volumes in the fore-bay and the storage reservoirs as the variables. Each combination of these two volumes is checked for feasibility and is denoted by a feasible cell, if the constraints are all satisfied and there is at least one pair of feasible discharges to reach a feasible cell at the next instant. From every feasible cell at the beginning of the N-th subinterval to reach the specified final state, the possible discharge rates and the corresponding weighted outputs are calculated and recorded. This results in a table for the N-th instant giving the feasible cells with relevant information recorded in them. Similarly tables are constructed for all instants $i = 1,\ldots,N$, working from N-th instant backwards. From the tables so constructed, the optimum schedule can be obtained by identifying the best possible route. The procedure, as can be seen, becomes quite unwieldy if there is more than one pumped storage station in the power system and the computational storage and time requirements will also become quite high. Bernard etal [38] have considered one

pumped storage plant in conjunction with a thermal plant and suggested an iterative procedure of obtaining the optimum pumping generating schedule starting with a nominal schedule. Their procedure is based on the following two results obtained using the calculus of variations;

(i) Optimum utilization of the available water is obtained when the hydro generation is scheduled for specified subintervals such that the incremental fuel savings are equal for each corresponding subintervals,

(ii) Additional economy may be obtained as long as the incremental fuel saving exceeds the incremental pumping cost. To start with, incremental pumping cost curves as well as incremental fuel saving cost curves are determined for each of the pumping as well as the generating subintervals (prespecified) respectively as a function of volumes of water. Starting from a nominal schedule of pumping and generating, the various incremental fuel saving costs and the incremental pumping costs for the corresponding subintervals are obtained from the above curves. The method consists of systematically improving the hydro schedule ensuring the adherence to the above two results. The iterative process converges to the optimum schedule when the incremental fuel savings cost is equal to the incremental pumping cost for each subinterval in the pumping phase. This procedure becomes difficult for a problem consisting of more than one pumped storage and one thermal plants, as the fuel

saving cost and pumping cost curves for the corresponding subintervals as a function of water volumes cannot be independently obtained as in the simple case seen above. Further in their model, the transmission losses in the system are not considered.

Brainbridge etal [39] used a gradient procedure on the hydro system of the problem and the dynamic programming procedure combined with the use of coordination equations for the thermal system. The basic procedure in this method is to start with a trial schedule, and by an iterative adjustment of the outputs of the hydro plants in the direction of steepest descent, the corresponding outputs of the thermal plants are obtained using the combination of dynamic programming procedure and the solution of the linear coordination equations. The iterative process is repeated until a minimum cost is attained. Cobian [40] considered the problem of a pumped storage plant with three interconnected power systems. The problem is to determine the optimal power interchanges between the different power systems and the pumped storage plant. Each power system is represented by its cost characteristic and the pumped storage plant by its reservoir dynamics. Transmission losses are neglected in the formulation (however, he has suggested an approximate way of accounting for the transmission losses in the interconnected power system). Dynamic Programming procedure

is used to obtain an optimum schedule. In each subinterval of optimization, a number of mathematical programming problems are to be solved and the modified version of Rosen's [44] gradient projection technique is employed to solve these mathematical programming problems. As admitted by the author in his paper [40], this procedure is not applicable to systems having more than two pumped storage plants due to excessive fast access computer memory requirements. Rees and Larson [41] have also considered a pumped storage plant in combination with an interconnected power system. The diversity exchange contracts also are considered in the formulation of the problem and the transmission losses are neglected. Successive Approximations to Dynamic Programming (S.A.D.P.), [32] is employed to solve the above problem.

In all the above attempts, it can be seen that basically Dynamic Programming with some modifications in certain cases has been employed in solving the pumped storage scheduling problem under consideration. Because of the very nature of the solution procedure requiring excessive core storage and computational time, the attempts have been limited to small systems. In this chapter, the Methods of Local Variations and the Optimal Power Flow used in Chapter 5 have been employed again to solve the problem under consideration. The superiority of the basic approach employed here regarding the computational

time and core storage requirements has already been shown in Chapters 4 and 5. The inclusion of the pumped storage plants does not alter the claims made previously regarding the computational core storage and time requirements.

6.2 PROBLEM FORMULATION

The optimal scheduling problem in a power system consisting of pumped storage plants in addition to conventional hydro plants and thermal plants is formulated as a discrete time optimal control problem as follows. Let p,h and r represent the number of pumped storage, conventional hydro and the thermal plants respectively. Let $x_i(k)$ be the storage in the reservoir of the i-th hydro plant (in the case of a pumped storage plant, it refers to the storage of the fore-bay reservoir) at the beginning of the k-th subinterval. Similarly let $u_i(k)$ be the discharge rate from the reservoir of the i-th hydro plant (in the case of a pumped storage plant, this refers to the pumping rate of water from the back-bay reservoir into the fore-bay reservoir). The total interval of optimization is assumed to be divided into N equal subintervals of unit length for simplicity (as in Chapters 4 and 5). The initial and final storages of the reservoirs in the pumped storage as well as conventional hydro plants are assumed to be specified. The inflows into the reservoirs of the pumped storage plants are neglected in this formulation without loss of generality. The hydro subsystem dynamics is defined by the equations

$$x_i(k+1) = x_i(k) - u_i(k) \ , \ (i=1,\ldots,p), \ (k=1,\ldots,N) \qquad (6.1)$$

$$x_i(k+1) = x_i(k) + L_i(k) - u_i(k), \ (i=p+1,\ldots,p+h), \ (k=1,\ldots,N) \qquad (6.2)$$

$x_i(1)$ and $x_i(N+1)$, $(i = 1,\ldots,p+h)$ are specified. It may be noted here that $u_i(k)$ is negative in the pumping mode of the plant and positive in the generating mode. The pumping and generating powers can be approximately related to the water head, discharge rate (or pumping rate) by the following relations.

Power required during the pumping operation, by the i-th pumped storage plant in the k-th subinterval is

$$P_{Hi}(k) = \frac{H_{oi}\ \eta_p}{G}[1 + \frac{c_i}{2}(x_i(k) + x_i(k+1))]u_i(k) \qquad (6.3)$$

and power generated by the i-th conventional hydro or pumped storage plant during the generation operation in the k-th subinterval is

$$P_{Hi}(k) = \frac{H_{oi}\ \eta_G}{G}[1 + \frac{c_i}{2}(x_i(k) + x_i(k+1))]u_i(k) \qquad (6.4)$$

where η_p and η_G are the efficiencies of the plant in the pumping and generating modes respectively. It may be noted here that in eqn.(6.3), if $u_i(k)$ is negative (for pumping operation), then power $P_{Hi}(k)$, will also be negative. The discharge rates, pumping rates and the storages of the hydro reservoirs are bound by the following

upper and lower limits imposed by the operating and engineering constraints.

$$-u_{i\ max} \le u_i(k) \le u_{i\ max}\ ,\ (i=1,\ldots,p),\ (k=1,\ldots,N) \quad (6.5)$$

$$u_{i\ min} \le u_i(k) \le u_{i\ max}\ ,\ (i=p+1,\ldots,p+h),\ (k=1,\ldots,N) \quad (6.6)$$

$$x_{i\ min} \le x_i(k) \le x_{i\ max}\ ,\ (i=1,\ldots,p+h),\ (k=2,\ldots,N) \quad (6.7)$$

The formulation of the thermal subsystem and the electrical network is the same as in Sec. 2.3 given by eqns.(2.15) to (2.26). Now the optimization problem is to find the optimal schedules for the pumped storage plants (pumping and generating modes), conventional hydro plants and the thermal plants minimizing the fuel cost given by eqn.(2.15), while satisfying the various constraints (6.5) to (6.7), (2.16), (2.17) and (2.20) to (2.26).

6.3 DESCRIPTION OF THE ALGORITHM

Step 1: To start with, a nominal hydro trajectory of storages of the pumped storage as well as the conventional hydro plants over the entire interval of optimization is assumed and is tested for feasibility as in Sec. 5.4. During the above procedure the discharge as well as pumping rates and their corresponding powers are available for each subinterval for the nominal trajectory.

Step 2: Considering the hydro generation or the pumping power required as injections or loads at the appropriate buses, optimal power flow solutions are obtained one for each subinterval to determine the corresponding optimal thermal generations in the system. The subinterval costs as well as total cost are calculated and stored.

Step 3: The current nominal hydro trajectory is varied successively one hydro station after the other (including pumped storage plants) at all the instants k=2,...,N using the MLV as in Sec. 5.4 until a satisfactory convergence to the optimum solution is obtained.

It is to be noted here that in the variational process, if at any instant k, the varied state value $\tilde{x}_i(k) > \tilde{x}_i(k-1)$, the plant under consideration is in pumping mode, whereas if $\tilde{x}_i(k) < \tilde{x}_i(k-1)$, it is in generating mode.

6.4 NUMERICAL RESULTS

The configuration of the system considered for illustration here is the same as in Figure 5.1 except that the hydro station at bus 1 is assumed to be a pumped storage plant. The interval of optimization (one day) is subdivided into 6 subintervals, each of 4 hours duration as in Sec. 5.5. The line and load data are the same as in Tables 5.1 and 5.2 respectively. Table 6.1 gives the hydro subsystem characteristics. The cost coefficients of the thermal stations are the same as in Table 5.4.

Table 6.2 gives the upper and lower limits on the reservoir storages at the various instants over the optimization interval. Table 6.3 stipulates the limits on the voltages, active and reactive powers at the various buses. Table 6.4 gives the initial and optimal trajectories of the hydro reservoirs. Table 6.5 gives the optimal schedule for the pumped storage, conventional hydro and thermal plants obtained using the variable step size vector as in Chapters 4 and 5, with an initial step size of $\Delta x = 200$ meter^3/sec.-4 hours for both the pumped storage and the conventional hydro plants. Table 6.6 gives the voltages, phase angles and reactive powers at all the buses over the entire interval of optimization (for the optimum condition). The pumping and generating efficiencies η_P and η_G are taken as 0.86 and 0.84 respectively.

TABLE 6.1: CHARACTERISTICS OF THE HYDRO SUBSYSTEM

Hydro system	Pumped storage plant (i=1)	Conventional hydro plant (i=2)
Initial storage in meter^3/sec.-4 hrs.	2330	2330
Final storage in meter^3/sec.-4 hrs.	2330	2330
Maximum allowable discharge rate in meter^3/sec.	500	500
Minimum allowable discharge rate in meter^3/sec.	-500	100
Initial head of the reservoir in meters	14	14
Final head of the reservoir in meters	14	14
Basic head H_{oi}	10	10
Inflow (assumed constant throughout)	0	300

Note: Negative sign in the above table shows the pumping mode.

TABLE 6.2: UPPER AND LOWER LIMITS ON THE RESERVOIR STORAGES

Instant k	1	2	3	4	5	6	7
$x_{2\ max}$ in meter^3/sec.-4 hrs.	2330	2365	2400	2430	2400	2365	2330
$x_{2\ min}$ in meter^3/sec.-4 hrs.	2330	2300	2260	2230	2260	2300	2330
$x_{1\ max}$ in meter^3/sec.-4 hrs.	2330	2400	2460	2500	2450	2390	2330
$x_{1\ min}$ in meter^3/sec.-4 hrs.	2330	1800	1800	1800	1800	1800	2330

TABLE 6.3: LIMITS ON VOLTAGES AND GENERATIONS

Bus Number	VOLTAGE		REAL POWER		REACTIVE POWER	
	Upper limit (in p.u.)	Lower limit (in p.u.)	Upper limit (in p.u.)	Lower limit (in p.u.)	Upper limit (in p.u.)	Lower Limit (in p.u.)
1^+	1.02	1.02	1.1	0.1	0.6	-0.6
2^+	1.04	1.04	1.1	0.1	0.6	-0.6
3	1.10	0.9	0.0	0.0	0.0	0.0
4^+	1.02	1.02	1.1	0.1	0.6	-0.6
5^+	1.04	1.04	1.1	0.1	0.6	-0.6

Note: + marked buses in the above table are voltage controlled buses.

TABLE 6.4: INITIAL AND OPTIMAL TRAJECTORIES OF THE RESERVOIR STORAGES

Instant k	1	2	3	4	5	6	7
Initial storage $x_1(k)$	2330.0	2100.0	2280.0	1940.0	2080.0	2200.0	2330.0
Optimal storage $x_1^*(k)$	2330.0	2325.0	2380.0	2340.0	2080.0	2078.125	2330.0
Initial storage $x_2(k)$	2330.0	2328.0	2328.0	2328.0	2328.0	2328.0	2330.0
Optimal storage $x_2^*(k)$	2330.0	2362.375	2399.875	2428.0	2399.875	2362.375	2330.0

Note: The units are the same as defined earlier.

TABLE 6.5: OPTIMAL SCHEDULE FOR PUMPED STORAGE, HYDRO AND THERMAL STATIONS

Subinterval k	1	2	3	4	5	6
$P_{H1}(k)$ (Pumped storage plant)	0.0051	-0.0560	0.0417	0.2681	0.0187	-0.2535
$P_{H2}(k)$ (Conventional hydro plant)	0.2723	0.2677	0.2780	0.3355	0.3442	0.3381
$P_{T1}(k)$	0.6193	0.8897	1.023	0.7094	0.5693	0.7184
$P_{T2}(k)$	0.3130	0.5082	0.6685	0.5043	0.5000	0.2124
$u_1(k)$	5.0	-55.0	40.0	260.0	1.875	-251.875
$u_2(k)$	267.625	262.5	271.875	328.125	337.5	332.375

Note: In the above table a negative sign indicates the pumping operation during that subinterval. All the units are same as defined earlier except the discharges $u_1(k)$ and $u_2(k)$, which are given in meter^3/sec.

Initial cost in dollars = 11063.80

Final cost in dollars = 11018.08

TABLE 6.6: VOLTAGES, PHASE ANGLES AND REACTIVE POWERS AT THE VARIOUS BUSES

Sub-interval k	1	2	3	4	5	6
V_1	1.02	1.02	1.02	1.02	1.02	1.02
δ_1	0.358	-0.107	0.583	1.967	0.339	-1.237
V_2	1.04	1.04	1.04	1.04	1.04	1.04
δ_2	-0.180	-0.028	-0.025	-0.387	1.399	0.175
V_3	1.0275	1.0248	1.0219	1.0229	1.0256	1.0284
δ_3	-0.104	-0.495	-0.922	-0.659	1.182	0.698
V_4	1.02	1.02	1.02	1.02	1.02	1.02
δ_4	6.5	5.991	5.784	7.378	9.369	8.64
V_5	1.04	1.04	1.04	1.04	1.04	1.04
δ_5	0.0	0.0	0.0	0.0	0.0	0.0
Q_1	-0.1993	-0.1817	-0.2097	-0.2703	-0.1984	-0.1219
Q_2	0.1386	0.1465	0.1636	0.1815	0.1187	0.1378
Q_3	0.0	0.0	0.0	0.0	0.0	0.0
Q_4	-0.0747	-0.0673	-0.0616	-0.0716	-0.0793	-0.0847
Q_5	0.2926	0.2995	0.3504	0.4049	0.3576	0.2253

Note: $Q_1,\ldots,Q_5$ are the reactive powers at buses 1,...,5 respectively. All units are in p.u. except the phase angles (δ's), which are given in degrees.

6.5 DISCUSSION ON THE SERIES CONNECTED HYDRO RESERVOIRS

Case (i) Without Transport Delays:

In all the problems considered hitherto, in this thesis, the hydro reservoirs are considered to be independent of each other. The extension of the solution methods discussed so far to the case of series connected reservoirs is straightforward as can be seen in this section. Consider a power system with h hydro plants with a series operation of some of the reservoirs. The dynamics of the hydro system may be represented as

$$x_i(k+1) = x_i(k) + L_i(k) - u_i(k) + \sum_{j=1}^{h} \mu_{ij}\, u_j(k),$$
$$(i=1,\ldots,h),\ (k=1,\ldots,N) \qquad (6.8)$$

$$\text{where } \mu_{ij} = \begin{cases} 1 \text{ if reservoir } i \text{ is directly upstream from reservoir } j \\ 0 \text{ otherwise} \end{cases} \qquad (6.9)$$

The operating constraints remain the same as in the earlier cases as

$$u_{i\,min} \leq u_i(k) \leq u_{i\,max}\ ,\ (i=1,\ldots,h),\ (k=1,\ldots,N) \quad (6.10)$$

$$x_{i\,min} \leq x_i(k) \leq x_{i\,max}\ ,\ (i=1,\ldots,h),\ (k=2,\ldots,N) \quad (6.11)$$

The algorithms of Secs.4.7, 5.4 and 6.3 can be carried out with the following simple modification. In the variational process, let the state value corresponding to the i-th

reservoir at the instant k be under consideration. When $x_i(k)$ is incremental to $x_i(k) \pm \Delta x$, the discharge rates in the (k-1)-th and k-th subintervals are evaluated using (6.8) for the i-th as well as the reservoirs situated downstream to the i-th reservoir. If any of the discharge rates or the storages of the reservoirs under consideration violate the limits on them, the variation under consideration is declared infeasible and the process is repeated as in the earlier cases. The rest of the steps in the above algorithms remain unaltered.

Case (ii) With Transport Delays:

Let τ_j be the transport delay (which is assumed to be expressible in terms of integral multiple of the sub-interval duration), which elapses before the water released from the j-th reservoir reaches the i-th reservoir (downstream of j-th reservoir). Then the dynamics of the hydro system may be represented as

$$x_i(k+1) = x_i(k) + L_i(k) - u_i(k) + \sum_{j=1}^{h} \mu_{ij}\, u_j(k-\tau_j) ,$$

$$(i=1,\ldots,h), \quad (k=1,\ldots,N) \qquad (6.12)$$

where μ_{ij} is defined the same way as in (6.9).

In applying the MLV in this case with transport delays, the following modification is to be made in the basic algorithms described in Secs. 4.7, 5.4 and 6.3. Let the state value corresponding to the i-th reservoir

at the k-th instant be under consideration. When $x_i(k)$ is incremented to $x_i(k) \pm \Delta x$, the discharge rates in (k-1)-th and k-th subintervals along with discharge rates in the affected subintervals of the reservoirs which are downstream to the i-th reservoir are determined using (6.11). The limits on the storages and discharge rates corresponding to these subintervals are checked for the feasibility of the variation under consideration. The costs of these subintervals under consideration are determined (in the same way as in Secs. 4.7, 5.4 and 6.3) and compared with their costs before this variation is effected. Depending on the success or failure of this variation at the k-th instant, the process is advanced to the (k+1)-th instant. Rest of the steps remain the same as in the algorithms in Secs. 4.7, 5.4 and 6.3.

6.6 CONCLUSIONS

Numerical results are presented in Tables 6.1 to 6.6 for a 5 bus system (whose configuration is the same as in Figure 5.1) consisting of one pumped storage, one conventional hydro and two thermal stations using the MLV and the Optimal Power Flow method. The generality of the MLV in dealing with the pumped storage plants is established. All the engineering and operating limits on the hydro and pumped storage plants and the electrical network can be taken into account without any modification of the MLV and Optimal Power Flow algorithms discussed in Chapter 5.

A brief discussion on the consideration of the series connected reservoirs (without and with transport delays) in the optimal scheduling problem in hydro-thermal power systems is also included.

CHAPTER 7

CONCLUSION

7.1 INTRODUCTION

This chapter is aimed at reviewing the significant results obtained during the course of this work and making a few suggestions for the future line of research in this area. Before reviewing the work done, a brief account of the objectives of this investigation is summarized.

As is already mentioned in Chapter 1, the problem of optimal scheduling in hydro-thermal power system is more complex than in a purely thermal system, since the dynamics of the hydro reservoirs make the problem a variational problem, whereas a purely thermal scheduling problem is a static optimization problem. Several attempts have been made in the past to solve both the long range and short range optimal scheduling of hydro-thermal system problems using direct and indirect search methods. The application of direct search techniques like the Dynamic Programming [15], Incremental Dynamic Programming [10] etc. have resulted in solution procedures requiring large computational time and core storage. With larger inter-connected power systems, the procedures are hence hence inapplicable. Indirect search techniques using

the Pontriagin's maximum principle (both the continuous and the discrete versions) result in a Two Point Boundary Value Problem (TPBVP) to be solved iteratively. The lack of proper techniques for solving the above TPBVP satisfying the constraints on both the state and control variables make these methods applicable only to particular systems. Further the limits on the state variables present considerable difficulty in these methods.

Hence the main objective of the present work has been to find a suitable solution technique which can be applied to the optimal scheduling problem in larger interconnected hydro-thermal systems and which requires lesser computational time and core storage than the existing methods.

7.2 REVIEW OF THE WORK DONE

The proposed solution method in this thesis consists of a decomposition of the combined systems into hydro and thermal subsystems and starting from a nominal hydro trajectory, the optimal trajectory is obtained by iteratively varying the initial nominal trajectory by a direct search technique known as MLV [23], [24]. In each subinterval of optimization, a mathematical programming problem is to be solved for obtaining the optimal thermal generations. Augmented Penalty Function Method [26] or Optimal Power Flow Method is employed to solve the above problem encountered, depending on the model used.

The first part of the thesis is devoted to investigate the solution using an approximate model discussed in Sec. 2.2. In this model the electrical network is represented by the loss coefficients and the constraints on the voltage magnitudes, reactive powers and the line flows etc. have been ignored. To illustrate the method of solution a numerical example similar to the one considered in [10] has been solved. In a one-hydro-one-thermal problem considered here, the thermal generations are obtained by simply solving a quadratic equation. A detailed comparison of the solution algorithm based on MLV is made with that of IDP due to Bernholtz and Graham [10]. The motivation for this comparison is that both these techniques are direct search techniques in the neighbourhood of a nominal trajectory. The superiority of the proposed algorithm over IDP procedure from the aspect of core storage requirements is established. Then the procedure is extended to a multi-hydro multi-thermal optimal scheduling problem. A numerical example of three hydro and four thermal stations has been solved to illustrate the solution procedure. In each subinterval the mathematical programming problem encountered, is solved by the Augmented Penalty Function Method to solve for the optimal thermal generations. With the simplicity and speed of this method, the overall solution procedure has become simple and fast. The problems solved in this part show that the proposed method is quite promising

based on the experience gained during the course of this work, it is recommended to start the MLV algorithm initially with as large a step size vector as is consistent with the problem and gradually reduce it each time as the iterative process converges. This procedure has produced significant reduction in computation times, compared to working with a constant (small) step size vector. The reduction is more significant in the larger dimensional problems (Secs. 4.8, 5.5) compared to the one-hydro-one-thermal case.

To show the generality of the suggested solution techniques in the earlier parts, a general optimal scheduling problem in a system consisting of pumped storage, conventional hydro and thermal plants is considered in Chapter 6. The solution procedure does not need any modifications of the basic solution algorithms already employed in the earlier parts and the core storage and computational time requirements do not get altered significantly in this case.

To summarize, the application of the MLV to correct the initial nominal trajectory iteratively in combination with either the Augmented Penalty Function Method or the Optimal Power Flow Method affords a simple, efficient and easily implementable solution technique to the optimal scheduling problem in hydro-thermal power systems.

Based on the investigations made and the results obtained, the following major conclusions can be drawn.

(i) The algorithms proposed in Chapters 4 and 5, using the approximate and sophisticated models respectively, provide simple and efficient procedures of solving the optimal scheduling problem for hydro-thermal power systems. In both the cases all the constraints are efficiently taken care of. Both the methods are applicable to problems of large interconnected systems, as the computational time and core storage requirements of these methods are significantly less compared to those of the existing methods [10], [20] etc.

(ii) The variable step size procedure suggested, while working with MLV, affords a significant reduction in the computational time required than that of working with constant (small) step size throughout.

(iii) The inclusion of pumped storage plants in the proposed solution algorithms poses no difficulty.

(iv) The extension of the solution algorithms suggested need little modification to consider the series connection of the hydro reservoirs with and without transport time delays.

7.3 FUTURE LINE OF RESEARCH

As a sequel to the results obtained during the present investigation, the following line of future research seems to be worth pursuing.

1. In all the problems considered, the load demand on the system and the inflows into the reservoirs have been taken to be deterministically known in advance. But in actual practice, these can only be characterized probabilistically (particularly in the case of a long range scheduling problem) and hence the scheduling problem becomes a stochastic control problem. Suitable modifications of the solution algorithms proposed in this thesis may be possible to consider the stochastic nature of the load demand and the inflow data.

2. In the problems considered in Chapters 4 and 5, the objective function to be minimized and the equality constraint were nonlinear and hence nonlinear programming techniques involving substantial computational effort had to be employed. Simple linear programming problems would result by linearizing the objective function and the equality constraint around the operating point, which can be solved more easily. Since working with the linearized equations will ensure the satisfaction of the original constraints only in the neighbourhood of the operating point, reinitialization has to be done once after every few iterations. An attempt by the author in this direction for a one-hydro-one-thermal problem seems to be quite promising and it appears that the linearization may lead to considerable reduction in computational times for higher dimensional problems.

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